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by

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Solving and Interpreting Large-Scale Harvest Scheduling Problems by Duality and Decomposition

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ABSTRACT. This paper presents a solution to the forest planning problem that takes advantage of both the duality of linear programming formulations currently being used for harvest scheduling and the characteristics of decomposition inherent in the forest land class-relationship. The subproblems of decomposition, defined as the dual, can be solved in a simple, recursive fashion. In effect, such a technique reduces the computational burden in terms of time and computer storage as compared to the traditional primal solutions. In addition, utilization of this method allows the use of two simple procedures for creating an initial, basic, feasible solution. Forest management alternatives within one (or more) land class can be evaluated easily in this framework, and multiple-use considerations can be incorporated directly into the optimization as nonharvest values.

ADDITIONAL KEY WORDS. Duality, decomposition.

Solving and Interpreting Large-Scale Harvest Scheduling Problems by Duality and Decomposition*

The Forest Service uses linear programming to produce timber harvest plans consonant with the nondeclining flow requirement of the National Forest Management Act. Pioneered by Navon (1971), linear program scheduling techniques were improved by Johnson and Scheurman (1977) and incorporated into the Forest Service's current scheduling tool, FORPLAN (Johnson et al. 1980).[1] However, the scheduling problems solved by the Forest Service are larger and more complex than the problem described by Johnson and Scheurman (1977). The size of these problems makes their solution expensive (as much as $200,000 for a single forest) and their interpretation difficult. This paper exploits the structure of the basic scheduling problem to produce fast, compact computer code and to aid in the interpretation of optimal plans.

The problem solved in this paper is the efficient solution of Johnson and Scheurman's Model II scheduling problem with nondeclining flow. This problem is one of the many schedules developed for each forest for comparison purposes. The scheduling runs that produce the selected plans include all the constraints in this problem plus additional constraints on size of cuts, old growth retention, and others. Thus efficient solution of the Model II with nondeclining flow problem is interesting in its own right and will help in the ultimate efficient solution of larger problems.

Part of the difficulty in interpreting linear programming scheduling solutions is finding the effects of constraints such as nondeclining flow on the desirability of management techniques proposed for an individual stand. The algorithm presented in this paper provides a set of present value prices that can be used to check the desirability of management techniques other than the one selected by the linear program. For instance, the value of thinning commercial site class 144 douglas fir stands twice rather than once, can be computed by a few arithmetic calculations without rerunning the model. The same set of present value prices give an approximation of the cost of the constraints imposed on the model. These methods, although used to price non-declining flow in this model, are easily extended to any constraints and can used with the standard FORPLAN output.

This paper is presented in six sections. Following the Introduction, Section 1 shows how to solve Johnson and Scheurman's Model II, Form I, without using an iterative technique such as the revised simplex method.

Section 2 shows how to decompose the forest planning problem by using the algorithm of Dantzig (1963) and Dantzig and Wolfe (1961), which has been used in different forest-planning contexts by Nazareth (1973) and by Williams (1976).

Section 3 provides an extremely simple way to start the decomposition algorithm. Section 4 shows how decomposition and nondeclining flow shadow price can be used to evaluate alternative management strategies such as a pest-suppression program.

Section 5 extends the algorithm so that forest land is valued both for the timber cut and for the other resources produced. For instance, a manager might wish to maximize the value of timber plus the value of water. Section 6 provides a summary and discussion.

1. AN ANALYTIC APPROACH TO HARVEST SCHEDULING

Johnson and Scheurman's model (Model II, Form I; 1977, p. 5) captures the essence of the U. S. Forest Service's current linear programming approach to forest planning. Using their notation, the model is represented as:

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{ij}\, x_{ij} + \sum_{i=-M}^{N} w_{iN}E_{iN} \tag{1.1}$$

where x_{ij} is the harvest and D_{ij} is the value in year j of land planted in year i. N is the terminal time; hence, w_{iN} are the acres left uncut and E_{iN} is the value of the uncut acres in year N that were planted in year i. The regeneration time lag is z, and the oldest timber was regenerated at time −M.

The constraints are

$$\sum_{j=1}^{N} x_{ij} + w_{iN} = A_i \qquad i = -M, \ldots, 0 \tag{1.1a}$$

which states that the inventory of land planted at time i, A_i, is either cut or left standing and

$$\sum_{k=j+z}^{N} x_{jk} + w_{jN} = \sum_{i=-M}^{j-z} x_{ij} \qquad j = 1, \ldots, N \tag{1.1b}$$

which states that land harvested in year j from all previous plantings is available for harvesting between year j + z and year N or left standing, w_{jN}. The linear program is max equation (1.1) subject to equations (1.1a) and (1.1b) and to nonnegativity constraints.

Although Form I of Model II is well suited for the direct use of the revised simplex method, a noniterative solution requires a form with many more constraints. Define w_{it} as the acreage planted at year i available for harvest after date t.

$$w_{it} = \sum_{j=t+1}^{N} x_{ij} + w_{iN}. \tag{1.2}$$

Using this definition, equation (1.1a) is equivalent to

$$x_{i1} + w_{i1} = A_i \qquad i = -M, \ldots, 0 \tag{1.3}$$

and

$$w_{it} - w_{it+1} = x_{it+1} \qquad \begin{array}{l} i = -M, \ldots, 0 \\ t = 1, \ldots, N - 1. \end{array} \tag{1.4}$$

The indexing set, $I = [(i, j) | i = -M, \ldots, N; j = 1, \ldots, N]$, can be divided into two sets, T and T^c, their union being the whole of I:

$$T = [(i, j) \mid i = -M, \ldots, N; j = \max (1, i + z), \ldots, N]$$

$$T^c = [(i, j) \mid i = -M, \ldots, N; j = 1, \ldots, \max (1, i + z) - 1].$$

These sets can be used to separate meaningful harvests from empty harvest possibilities as shown in the following new constraint

$$x_{ij} = 0 \qquad \text{if } (i, j) \in T^c. \tag{1.5}$$

Now, repeatedly, apply equation (1.2) to equation (1.1b) to obtain

$$w_{jj-1} = \sum_{i=-M}^{j} x_{ij} \qquad j = 1, \ldots, N \tag{1.6}$$

and

$$w_{it} - w_{it+1} = x_{it+1} \qquad \begin{array}{l} i = 1, \ldots, N \\ t = i - 1, \ldots, N - 1. \end{array} \tag{1.7}$$

The extended form of Model II is to max equation (1.1) subject to equations (1.3) through (1.7) and to nonnegativity constraints.

The Lagrangian function for this problem is to be $\min_{\lambda\gamma} \max_{x,w} L$,

$$L = \sum_{i=-M}^{N} \sum_{j=1}^{N} D_{ij} x_{ij} + \sum_{i=-M}^{N} w_{iN} E_{iN} \tag{1.8}$$

$$+ \sum_{i=-M}^{0} \left\{ \lambda_{i1} (A_i - x_{i1} - w_{i1}) \right.$$

$$\left. + \sum_{t=1}^{N-1} (w_{it} - w_{it+1} - x_{it+1}) \lambda_{it+1} \right\}$$

$$+ \sum_{j=1}^{N} \lambda_{jj-1} \left(-w_{jj-1} + \sum_{i=-M}^{j} x_{ij} \right)$$

$$+ \sum_{i=1}^{N} \sum_{t=i-1}^{N-1} (w_{it} - w_{it+1} - x_{it+1}) \lambda_{it+1}$$

$$- \sum_{(i,j)\in T^C} \gamma_{it} x_{it}.$$

To solve this minimax problem (see Whittle 1971), recall that, at a saddle of L, the complementary slackness conditions are

$$L_{x_{ij}} \cdot x_{ij} = L_{w_{ij}} \cdot w_{ij} = 0,$$

where

$$L_{x_{ij}} = \frac{\partial L}{\partial x_{ij}}.$$

Making this substitution in L gives the dual problem

$$\min \sum_{i=-M}^{0} \lambda_{i1} A_i \tag{1.8a}$$

subject to

$$L_{x_{ij}} = D_{ij} + \lambda_{jj-1} - \lambda_{ij} \leq 0 \qquad (i, j) \in T \tag{1.8b}$$

$$L_{x_{ij}} = D_{ij} + \lambda_{jj-1} - \lambda_{ij} - \gamma_{ij} \leq 0 \qquad (i, j) \in T^c \tag{1.8c}$$

$$L_{w_{ij}} = -\lambda_{ij} + \lambda_{ij+1} \leq 0 \qquad i = -M, \ldots, N \quad j = \max(1, i-1), \ldots, N-1 \tag{1.8d}$$

$$L_{w_{iN}} = E_{iN} - \lambda_{iN} = 0 \qquad i = M, \ldots, N. \tag{1.8e}$$

These constraints can be rewritten as

$$\lambda_{ij} \geq [D_{ij} + \lambda_{jj-1} \text{ and } \lambda_{ij+1}] \qquad (i, j) \in T$$

$$\lambda_{ij} \geq [D_{ij} + \lambda_{jj-1} - \gamma_{ij} \text{ and } \lambda_{ij+1}] \qquad (i, j) \in T^c.$$

Since minimizing $\Sigma A_i \lambda_{i1}$ requires the minimum λ_{i1} and λ_{ij} is constrained recursively to be at least as large as λ_{ij+1} and $(D_{ij} + \lambda_{jj-1})$, it follows that

$$\lambda_{ij} = \max [D_{ij} + \lambda_{jj-1} \text{ and } \lambda_{ij+1}] \qquad (i, j) \in T. \tag{1.9}$$

In the case $(i, j) \in T$, γ_{ij} is positive, λ_{ij} will decrease if γ_{ij} increases; hence,

$$\lambda_{ij} = \lambda_{ij+1} \qquad (i, j) \in T^c. \tag{1.10}$$

Finally,

$$\lambda_{iN} = E_{iN} \qquad i = -M, \ldots, N. \qquad (1.11)$$

The solution to the dual problem is obtained by setting $\lambda_{iN} = E_{iN}$ and finding λ_{ij-1} from the λ_{ij} by rules (1.9) and (1.10).

The optimal program is constructed by using the complementary slackness conditions. Using the i^{th} class as an example, x_{i1}, the first period harvest is zero if $L_{x_{i1}}$ is nonzero. Equivalently, x_{i1} is zero if $\lambda_{i1} = \lambda_{i2}$. The remaining stock is zero if $L_{w_{i1}}$ is nonzero or $\lambda_{i1} = \lambda_{10} + D_{i1}$. These two rules determine whether acreage is cut or saved in period 1; they determine w_{i1}, the acreage available in period 2. Again allocating acreage to be harvested if $\lambda_{i2} = \lambda_{21} + D_{i2}$ and to be saved if $\lambda_{i2} = \lambda_{i3}$ gives the period 2 optimal program. This procedure is repeated until the program for all N periods is constructed.

Starting with the first period, one then constructs the program. The interpretation of the rules can be expressed as: If the value of stumpage, D_{ij}, and the regenerated stand, λ_{jj-1}, exceeds the value of the stand in the next time period, λ_{jj+1}, then the stand should be cut; otherwise, save the stand for harvest in a subsequent period.

2. DECOMPOSITION OF THE FOREST SCHEDULING PROBLEM

A common forest planning problem is to schedule the harvest on many land classes--each satisfying an area constraint--and, simultaneously, to consider the forestwide nondeclining flow. FORPLAN and similar methods solve this large linear programming problem with the revised simplex method. However, the method provides no added insight into the structure of the planning problem, and the costs in computer time and storage are large. By incorporating the dual methods of Section 2 into a Dantzig-Wolfe decomposition algorithm (Dantzig and Wolfe 1961, Dantzig 1963), this section develops a computational method that decreases markedly the storage required and elucidates the role of nondeclining flow shadow prices in forest planning.

Stating the problem formally, let there be S land classes, s, with harvested area, x^s_{ij}, remaining area, w^s_{ij}, and initial area, A^s_i. Define the volume harvested from land class s in period t as

$$h^s_t = \sum_{i=-M}^{t-z} v^s_{it} \, x^s_{it} \qquad \text{for } s = 1, \ldots, S \tag{2.1}$$

where v^s_{it} is the volume per acre in year t on land of class s planted in year i. Let G be the $(N - 1) \times N$ matrix.

$$G = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix}.$$

Forestwide nondeclining flow requires harvest to be nondecreasing over time. Hence, the master problem constraint,

$$\sum_{s=1}^{S} Gh^s + u = 0, \tag{2.2}$$

where h^s is the column vector with elements h_t^s and u is an N − 1 element vector of slack variables. Letting p_t be the present value price in year t, the forestwide planning problem is

$$\max_{h,x} \sum_{s=1}^{S} \sum_{t=1}^{N} p_t h_t^s, \tag{2.3}$$

subject to equation (2.2) and, for each s, to equations (2.1), (1.1a), and (1.1b), and to nonnegativity.

Instead of using the revised simplex method to solve equation (2.3), one can decompose it into a master problem with S subproblems. The solution proceeds (from the initial master basis described in Section 3) by solving the master problem and using its shadow prices in the formation of the objective functions of the subproblems. The subproblems are then solved, and their new plans, h_t^s, are added to the prior set of plans. The master problem involves the choice of a new, basic, feasible solution from the candidate plans. This procedure is repeated until convergence is achieved.

Starting with the subproblems at the beginning of the k^{th} iteration, one examines the master problem to find an objective function (D_{it}^s, E_{iN}^s) for each subproblem. These subproblems, solvable by the methods of Section 1, are

$$\max \sum_{j=1}^{N} \sum_{i=-M}^{j-z} D_{it}^s \, x_{it}^s + \sum_{i=-M}^{N} w_{iN}^s \, E_{iN}^s, \tag{2.4}$$

subject to constraints (1.1a) and (1.1b). One then calculates the harvest volume, h^s, by equation (2.1); the present discounted value of the current schedule, f^s, is calculated by

$$f^s = \sum_{t=1}^{N} p_t h_t^s. \quad (2.5)$$

The subproblem value $(h^s)_k$ and $(f^s)_k$ are returned to the master problem. (The parentheses and subscripts denote that this is the subproblem's communication at the k^{th} iteration.)

Using the calculations of harvest volume and present discounted value from the subproblems, the master problem is to find the linear combination of subproblem plans that meets the nondeclining flow constraints and maximizes the present value of the plan. The k^{th} iteration begins by combining the previous optimal basis at the $k - 1^{th}$ iteration, M_{k-1}, with the S new candidate plans communicated by the subproblems at the end of the $k - 1^{th}$ iteration. The master problem chooses linear combinations of plans and uses index sets, R_k^s, to keep track of them[2]

$$R_k^s = \left[m \mid (h^s)_m \in M_{k-1}\right] \bigcup \left\{k - 1\right\}, \quad (2.6)$$

The master problem at iteration k is then defined by maximizing

$$\sum_{s=1}^{S} \sum_{m \in R_k^s} (f^s)_m \alpha_m^s \quad (2.7)$$

subject to:

$$\sum_{s=1}^{S} \sum_{m \in R_k^s} G(h^s)_m \alpha_m^s + u = 0 \quad (2.8)$$

and

$$\sum_{m \in R_k^s} \alpha_m^s = 1 \qquad \text{for } s = 1, \ldots, S. \quad (2.9)$$

At the k^{th} iteration, an optimal, basic, feasible solution to the master problem will consist of (N - 1 + S), nonzero α_m^s's, and N - 1 nondeclining flow shadow prices expressed in vector notation as

$$\pi = (\pi_1, \ldots, \pi_{N-1}).$$

Solution of the master problem at the k^{th} iteration yields new values for D_{it}^s for each s calculated as

$$D_{i1} = (p_{i1} + \pi_1)\, v_{i1}$$

$$D_{it} = (p_{it} - \pi_{t-1} + \pi_t)\, v_{it} \qquad t = 2, N - 1$$

$$D_{1N} = (p_{iN} - \pi_{N-1})\, v_{iN}$$

Those values are transmitted back to the subproblems, and the subproblems are solved. The iterative procedure ends when the matrix of candidate columns generated by the subproblems at some iteration contains no columns not already included in the basis of the master problem.

3. STARTING THE ALGORITHM

The solution algorithm described above is relatively easy to manipulate once an initial, basic, feasible solution to the problem is formulated. This section of the paper describes procedures for the development of that solution. In general, it is possible to find such a solution by a traditional Phase I procedure or other methods that consider the whole linear program. However, such a procedure requires simultaneous use of all of the subproblem and master-problem constraints and would incur all of the "setup" costs associated with a large, complete linear program.

Those costs can be avoided by noting that any set of subproblem harvest plans that meets the nodeclining flow requirements individually will also meet them collectively. Thus, solving each subproblem individually under non-declining flow constraints yields the initial basic, feasible solution for the master problem. An even less time-consuming method involves the construction of a set of subplans that harvests only at N (the end of time) or only at N and N - 1, or N, N - 1, N - 2, etc. For example, the following rules meet all area, as well as nondeclining flow, constraints on a stand-by-stand basis:

$$\forall_i, \text{ let } \; x_{iN} = A_i, \text{ all other } x_{it} = 0, \tag{3.1}$$

or

$$x_N = \frac{A_i \; v_{N-1}}{v_N + v_{N-1}}$$
$$x_{N-1} = \frac{A_i \; v_N}{v_N + v_{N-1}} \tag{3.2}$$

all other $x_{it} = 0$.

For the simple case under consideration, these methods provide an easy way to start the algorithm. Some, but not all, other constraints can be handled as well. For instance, cutting everything in the last period will meet an old growth retention constraint. An even-flow harvest plan might meet an overall budget constraint, but there is no guarantee that it would.

4. EVALUATING ALTERNATIVE MANAGEMENT STRATEGIES

Determining the economic benefits of a forestry practice, such as insect suppression or intensive forest management (Tedder and Schmidt 1980) in a planned forest requires the new practice to be evaluated at prices that reflect both market value of the timber output and the binding constraints of the harvest schedule. Dantzig and Wolfe (1961) have shown that a subproblem feasible plan that increases the value of the subproblems objective function will also increase the value of the overall objective function. This theorem provides a method to determine whether a new practice should be incorporated, at some level of intensity, into an optimal forest plan.

To be specific, consider undertaking an insect suppression program (e.g., spraying Bacillus Thuringiensis) on acres available for planting in year i that costs (present value) C per acre and increases yields from v_{it} to v_{it}'. The value of a plan without suppression, called the old plan, is $A_i \lambda_i$. The value of the new practice is found by solving the subproblem using the modified objective function from the old optimal solution:

$$\max \sum_{t=1}^{N} (p_t - \pi_t + \pi_{t-1})\, v_{it}'\, x_{it}, \tag{4.1}$$

subject to equations (1.1a) and (1.1b); that is, maximize present value using the subproblem objective function, which is adjusted for the value of the nondeclining flow constraints.

The addition and subtraction of shadow prices from the present value price reflects the costs of the nondeclining flow constraints. When both π_t and π_{t-1} are not zero, the nondeclining flow constraints are binding; and it is profitable to shift output from period t + 1 to period t and from period t to period t - 1. Of course, the nondeclining flow constraints prevent such a rearranging of the output stream. However, a suppression program that increases output at time t would make it possible to shift some output to time t - 1 without violating the nondeclining flow constraint. So, in addition to the market price of output, p_t, output at time t is worth π_{t-1} because it allows rearranging the output stream to get greater output at time t - 1. Similarly, increasing output at t requires rearranging the output stream to get greater output at t + 1. This rearrangement is not profitable, so π_t must be subtracted from the output price.

The new program is undertaken (at some positive level) if its value exceeds the value of the old program and the costs C. In the case of a spray program, this guarantees that some spraying would be optimal, but it does not guarantee that spraying all the eligible acreage will be profitable. Conversely, if the new program fails this test, it guarantees that none of the acreage should be sprayed. An analyst could confirm this theorem by rerunning the entire forest planning model allowing the scheduling model to choose whether or not to suppress the pest. Conversely, when this nondeclining flow adjusted cost-benefit condition does not hold, there is no point in rerunning the entire harvest schedule because the optimal level of suppression will be zero. Thus, pricing new alternatives with the modified objective function identifies unprofitable alternatives without the expense of rerunning the entire harvest scheduling model.

A more negative view of this theorem is that new plans that appear profitable when valued at market prices (the standard benefits less costs criteria) do not necessarily get incorporated into optimal forest plans. The reason is that such plans do not account for the nondeclining flow constraints.[3]

5. EXTENSIONS

So far, the objective function has only included timber harvest. More complicated forest scheduling problems require inclusion of either constraints or prices for other forest resources. Suppose, for instance, that water is produced by the forest in proportion to the amount of land in certain age and site categories. One can either add a constraint to the master problem to require a certain water flow or one can add a water price to the objective function to reflect the value of water. This section examines the latter method of valuing nontimber resources.

For example, let E_{it} $(t < N)$ represent the value of the flow of wildlife, water, aesthetic services, etc., generated in, upon, and around an acre regenerated in year i. Land costs associated with fertilization, fire protection, and other objectives could also be included in E.

The single land-class problem then would still be

$$\max \sum_i \sum_t D_{it}\, x_{it} + E_{it}\, w_{it},$$

subject to equations (1.3) through (1.7).

Retracing the steps in Section 1, one would obtain conditions (1.8a) through (1.8e). However, condition (1.8d) would be replaced with

$$L_{w_{ij}} = E_{ij} - \lambda_{ij} + \lambda_{i,j+1}. \tag{5.1}$$

The solution to the single land-class subproblem would then be

$$\lambda_{ij} = \max \left\{ D_{ij} + \lambda_{j,j-1} \quad \text{and} \quad E_{ij} + \lambda_{i,j+1} \right\} \qquad (i, j) \in T \tag{5.2}$$

$$\lambda_{ij} = \lambda_{i,j+1} + E_{ij} \qquad (i, j) \in T^C. \tag{5.3}$$

The variables, x and w, are then computed from complementary slackness conditions as before; and the results are used in the decomposition of Section 2.

6. CONCLUSIONS

The dual and decomposition methods reported in this paper provide a computational method to solve the harvest scheduling problem and a way to interpret the output of the currently used scheduling models.

The dual decomposition algorithm reported here was used to solve a medium-size scheduling problem constrained only by nondeclining flow and acreage and regeneration constraints. The sample problem had 12 stands split among three land classes. The time period included 15 decades before the schedule and 20 decades for the harvest schedule. On a Chi 2130 computer (roughly twice as fast as a Z80a based microcomputer with a hard disk), it took 75 minutes to solve the problem running a revised simplex algorithm provided by IBM (product H20-0238-2). Using the dual decompostion methods, the solution took only 37 minutes. The difference in speed may be attributable to differences in the codes other than the difference in the algorithm. Similarly for our machine and packages, the largest problem that could be accommodated by the revised simplex was about twice the size of the reported problem. The dual decomposition does not have that limitation, but the quality of its code was such as to preclude very large problems. The method is most attractive where the number of constraints (other than areal and regeneration constraints) is small, the number of land classes is large, the number of stands is even larger, and the machine resources are quite limited. Using a microcomputer to test the effects of insect suppression programs on nondeclining flow benchmark runs would be such a use. Of course, where as in classroom exercises, there are no constraints other than those of Model II Form I, the harvest schedule can be found with pencil and paper using the methods of Section 1.

In addition, this method provides a unifying framework for evaluating, on a forestwide basis, many forest management options--especially those that impact harvest yields over time. We have suggested only two such options: intensive management and insect control programs. Other management options that would impact yield are apparent and could be considered. Although the observation has been made previously [c.f. Binkley (1980) and selected references therein], we show that management options which change yield in a forest managed for nondeclining flow should be evaluated in terms of the non-declining flow shadow prices as well as in terms of their net value computed at market prices. Here, we extend the usefulness of that observation by providing a simple structure for evaluating such management options within one or more land classes in terms of their forestwide impact. Standard linear programming output includes the shadow prices of active constraints. These shadow prices can be used to form the decomposed objective function for each land class and the methods of Section 4, then suffice to evaluate the new plan. Similarly, just computing the decomposed objective function gives insight into the effect of the constraints. For instance, if the value of the decomposed price were only a fraction of the market price, then one could conclude that the constraints removed almost all of the economic incentive for forestry. In these circumstances, intensive forestry techniques that would be selected without the constraints are no longer selected. The advantage of these methods is that they properly account for the forestwide constraints and do not require the rerunning of the harvest schedule problem.

In conclusion, this paper provides a method to solve harvest scheduling problems when machine resources are limited and the number of constraints other than those that describe the growth of trees is not too large. The paper also provides a way to interpert existing harvest schedules and to evaluate new programs based upon their shadow prices.

FOOTNOTES

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[1] FORPLAN schedules harvests but does not allocate land to different uses such as commercial and noncommercial. MUSYC (Johnson and Jones 1980) allocates land and then schedules harvests on the commercial portion.

[2] The indices of the starting basis are nonpositive.

[3] Binkley (1980) presents, in a different context, a simple analysis of the role of nondeclining flow shadow prices for management that alters yield. He uses a cost-benefit framework that addresses some of the issues addressed here.

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