Title
Improving Critical Loss Analysis

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Market definition analysis, which is often central in merger cases, usually claims to follow the 1992 *Horizontal Merger Guidelines* issued by the U.S. Department of Justice and the Federal Trade Commission ("*Guidelines*").[1] The *Guidelines* describe a relevant product market as a group of products for which a hypothetical monopolist would profitably impose a "small but significant and non-transitory increase in price" ("SSNIP"). Seeking relatively simple approaches to market definition that are consistent with the *Guidelines*, courts and agencies often rely on Critical Loss Analysis.[2]

For example, the FTC recently challenged the proposed merger between Whole Foods and Wild Oats, two chains of grocery stores,[3] alleging that the relevant market was "premium natural/organic supermarkets" ("PNOS"). In that market, the merger was very highly concentrating in a number of geographic areas where Whole Foods and Wild Oats operated nearby stores. But the merging parties successfully argued that PNOS was too narrow a grouping of products and that the relevant market included all supermarkets. In that broader market, there were many other competitors and the merger was not highly concentrating.

Arguing that PNOS was not a relevant market, the merging firms echoed the Guidelines by asking whether a hypothetical PNOS monopolist would find a SSNIP profitable, or whether a SSNIP would deter enough sales to make it unprofitable. Critical Loss Analysis calculates the

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[1] A distinguished antitrust economist and lawyer begins a recent article by noting: "Throughout the history of U.S. antitrust litigation, the outcome of more cases has surely turned on market definition than on any other substantive issue." Jonathan Baker (2007), "Market Definition: An Analytical Overview," *Antitrust Law Journal*, 74: 129-173.

[2] The term "Critical Loss" was introduced by Barry Harris and Joseph Simons, (1989), "Focusing Market Definition: How Much Substitution is Necessary?" in *Research in Law and Economics*, Richard Zerbe, Jr. ed.

[3] Federal Trade Commission v. Whole Foods Market and Wild Oats Market, 502 F. Supp 2d 1, District Court for the District of Columbia, decided August 16, 2007. For a discussion of *Whole Foods* expressing skepticism about the FTC's case, see Carlton Varner and Heather Cooper, "Product Markets in Merger Cases: The *Whole Foods* Decision, *Antitrust Source*, October 2007.

hypothetical monopolist's Critical Loss, meaning the magnitude of lost sales that would (just) make it unprofitable for the hypothetical monopolist to impose a SSNIP, and compares it against the so-called Actual Loss of sales that would result from the SSNIP. If the Actual Loss would be less than the Critical Loss, the SSNIP would be profitable, so PNOS would form a relevant market. Whole Foods and Wild Oats argued that the Actual Loss would instead exceed the Critical Loss: a hypothetical PNOS monopolist who imposed a SSNIP would lose enough business to make the SSNIP unprofitable. Merging parties have used Critical Loss Analysis regularly, and with considerable success, to argue in court for a broader market than the government asserts.[4]

Estimating a hypothetical monopolist's Actual Loss is difficult, so that a substantial range of estimates could seem plausible. Incentives in litigation may push parties toward exploiting that range. Thus it is highly desirable, if possible, to anchor estimates of Actual Loss and to facilitate reality checks based on actual pre-merger conduct. When it comes to demand responsiveness, economics suggests that it is particularly helpful to examine firms' own pre-merger pricing conduct. It has been suggested, however, that pre-merger pricing is so remote from the hypothetical monopolist question that these reality checks are unhelpful. In this paper we examine that claim. We find that leading suggestions of how pre-merger pricing may be uninformative about a hypothetical monopolist's incentives are not compelling.

As a result, we are able to offer two powerful new tests to determine, using Critical Loss Analysis, whether a candidate group of products forms a relevant market.[5] These tests extract information from the gold standard for evidence about competitive conditions in antitrust cases: firms' actual business decisions made in the normal course of business.

1. A Short Refresher Course in Critical Loss Analysis

Suppose that a group of products, such as "PNOS," is proposed as a candidate market. How would one know, following the *Guidelines*, whether a hypothetical monopolist would find it profitable to impose a SSNIP? Simple economics tells us that the answer is "it would be profitable if demand is not too elastic," but that may not be very helpful.

Critical Loss Analysis contributes by asking quantitatively just *how* elastic demand must not be, for the candidate market to be a relevant market. This part of the analysis yields a number for the Critical Loss. It is often described as "just arithmetic," although as we shall see that is a

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[4] Michael Katz, (2002), "Recent Enforcement Actions by the U.S. Department of Justice: A Selective Survey of Economic Issues," *Review of Industrial Organization*, (2002), explains how Critical Loss Analysis was used by the merging parties in U.S. v. SunGard Data Systems, Inc. and Comdisco, Inc. 172 F. Supp. 2d 172 (2001). Additional such cases are described in Daniel O'Brien and Abraham Wickelgren (2003) "A Critical Analysis of Critical Loss Analysis," 71 *Antitrust Law Journal* 161.

[5] In this paper we follow the *Guidelines* methodology largely without question. In ongoing work, Joseph Farrell and Carl Shapiro, "Mergers with Unilateral Effects: A Simpler and More Accurate Alternative to Market Definition," we criticize the market definition approach and offer an alternative in cases involving unilateral effects. This paper will be available at http://faculty.haas.berkeley.edu\shapiro.

simplification. The meat of the analysis then involves estimating the Actual Loss—by no means a simple task. We may have no recent experience with a monopolist over just that group of products, and typically no business plans will exist for such a hypothetical monopoly. On the consumer side, while marketing studies in the ordinary course of business may well ask customers "If product X were unavailable, or more expensive, what would you buy instead?", they seldom ask "If products X, Y, and Z were unavailable, or more expensive, what would you buy?"

In practice, Critical Loss Analysis typically assumes that the products are symmetric in price and cost, and studies only a uniform SSNIP imposed on all products.[6] The assumption of product symmetry will, of course, seldom be correct, and a hypothetical monopolist might well want to raise some prices much more than others. In our companion paper we develop an alternative technique that is better suited to analyzing product asymmetries or non-uniform price increases.[7] Here we adopt the conventional simplification of symmetric products and a uniform SSNIP. We also make the standard assumption in Critical Loss Analysis that the hypothetical monopolist produces at constant marginal cost.

Let the pre-merger price be p, and the constant marginal cost be c, so the pre-merger gross margin is $m \equiv (p-c)/p$. Let the size of the SSNIP (as a percentage of pre-merger price) be s; following the *Guidelines*, we often illustrate with a 5 percent SSNIP, so $s = .05$. Let the pre-merger unit sales in the candidate market be X.

A. Critical Loss Arithmetic and Demand Sensitivity to Price Increase

Critical Loss analysis begins by calculating the Critical Loss, i.e., the (proportional) loss of sales that would just make a SSNIP unprofitable. In this calculation, the hypothetical monopolist's profit function is normally simplified to include only the direct, immediate, and quantifiable costs and revenues, marginal cost is simplified to be constant, and the SSNIP is simplified to be uniform. Using this simple proxy, the Critical Loss measured as a fraction of the hypothetical monopolist's pre-merger sales is given by $s/(m+s)$.[8] For example, with a margin of 45% and a SSNIP of 5%, the Critical Loss is $5/(45+5) = 1/10$, or 10%. The SSNIP will be profitable if and only if the actual loss of sales, as a fraction of pre-merger sales, is less than the Critical Loss.

This calculation asks: "Would a hypothetical monopolist find a SSNIP more or less profitable than the status quo?" This "break-even" version of the hypothetical monopolist's pricing incentives fits naturally with the separation of Critical Loss and Actual Loss calculations, and the

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[6] The asymmetric case can be analyzed, with additional complexity. See, for example, Serge Moresi, Steven Salop, and John Woodbury, "Implementing the Hypothetical Monopolist SSNIP Test with Multi-Product Firms," December 2007.

[7] Joseph Farrell and Carl Shapiro, "Cannibalization, Pass-Through, and Market Definition," uses a different technique and provides additional tests for market definition, also following the *Guidelines*, that allow for asymmetry among the products in the candidate market. This paper will be available at Shapiro's web site, http://faculty.haas.berkeley.edu/shapiro/.

[8] All technical derivations are in the Appendix.

approach been accepted by a number of courts, including the *Whole Foods* court, so we pursue it in Sections 1 to 5 below. The *Guidelines* actually ask the related but distinct question: "Is the hypothetical monopolist's *profit-maximizing* price at least a SSNIP above the status-quo level?" [9] The 1984 Guidelines did not explicitly mention a hypothetical profit-maximizing firm, referring instead to whether the SSNIP was "profitable." When the Critical Loss methodology was introduced in 1989, it was arguably consistent with the 1984 Guidelines. Its usage in the Courts has continued even though it has been inconsistent with the *Guidelines* for the past fifteen years. We discuss the relationship between the breakeven and profit-maximizing approaches, and modify our analysis to handle the profit-maximizing approach, in Section 6.

B. Estimating Actual Loss: The Role of Pre-Merger Margins

The second step in Critical Loss analysis is to estimate the number of lost sales, *Z*, or equivalently the fraction of pre-merger sales, Z/X, that will be lost due to a SSNIP.[10] We define the Actual Loss as this fraction: $L = Z/X$. Estimating the Actual Loss requires evidence about buyer substitution patterns. The controversy over Critical Loss Analysis concerns where to look for such evidence—specifically, how much can be inferred from pre-merger margins.

Katz and Shapiro (2003) and O'Brien and Wickelgren (2003) (hereafter KSOW) have shown how to estimate Actual Loss using information about demand responsiveness based on firms' pre-merger pricing decisions. This is what economists call "revealed preference" information: inferences about preferences based directly on observed choices. Here, one can make inferences about demand sensitivity, as gauged by a real firm based on its pre-merger choice of price. In particular, if (before the merger) a firm chooses a high margin on its product, the firm evidently thinks that demand for its product is not very sensitive to price.

This idea is captured in the Lerner Equation, the textbook centerpiece of pricing economics. The Lerner equation states that, if a product is priced so as to maximize the profits from that product, then the proportional gross margin, *m*, will be the inverse of the elasticity of demand facing the product, ε: that is, $\varepsilon = 1/m$.[11] For example, a margin of 50% (one-half) will be chosen when the firm faces an elasticity of two.

The Lerner Equation directly implies a prediction of the sales that would be lost by a single product if its price were to rise by a SSNIP. By the definition of elasticity, the fraction of sales

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[9] Section 1.0 defines a market as "a product or group of products and a geographic area in which it is produced or sold such that a hypothetical *profit-maximizing* firm, not subject to price regulation, that was the only present and future producer or seller of those products in that area likely would impose at least a 'small but significant and nontransitory' increase in price, assuming the terms of sale of all other products are held constant" (emphasis supplied). Section 1.11 states that "In performing successive iterations of the price increase test, the hypothetical monopolist will be assumed to pursue *maximum* profits in deciding whether to raise the price at any or all of the additional locations under its control." (emphasis supplied)

[10] Confusingly, the term "Actual Loss" means the *predicted* loss that would result from a *hypothetical* monopolist's SSNIP. Unfortunately, the term has become standard.

[11] A modified version of the Lerner Equation applies if the price affects the firm's profits in ways not captured by the margins the firm earns on this product alone; see Section 4.

lost is approximately εs,[12] and by the Lerner Equation, that in turn equals s / m. With a profit-maximizing margin of 45%, a single product whose price rises by a 5% SSNIP can be expected to lose a fraction 5/45, or 11.1%, of its sales.

Can one use the Lerner Equation to arrive at a reliable estimate of the hypothetical monopolist's Actual Loss in Critical Loss Analysis?[13] In stark form, the contending positions are:

- *Stress Pre-Merger Pricing Evidence*. If we want to know how the sales of a product, or a group of products, would respond to a price change, the best evidence normally is what the firms owning those products think, and what they do when their money is on the line. These firms know the market for their products better than any outside expert or court is likely to, and decisions taken in the normal course of business are less likely to be biased than is analysis developed in an adversarial setting. Economists generally treat revealed-preference information, i.e., direct inferences from actual choices, with special respect. Economics can fairly reliably infer a firm's beliefs about its demand elasticity from the firm's pre-merger behavior, specifically, the price that it chose for its product. Other information on demand responsiveness that was known to the firm should already be factored into its choice of price, and thus should be given little or no separate weight. Information on demand responsiveness that was available to the firm, but that the firm chose not to use, is probably not all that valuable, as judged by the firm when it had money on the line. Thus, at a minimum, the best simple way to estimate Actual Loss is to extrapolate, using economic logic, from the firms' pre-merger pricing choices.

- *Ignore Pre-Merger Pricing Evidence*. The use of revealed-preference information in this context is badly flawed for several reasons. Firms do not always maximize profits. Even if they do, maximizing profits in the real world is very complex, often requiring the firm to take into account spillovers to its other (substitute or complement) products, but also involving customer loyalty, reputation, learning-curve effects, network effects, and so on. These factors are not included in any practicably implementable form of the Lerner Equation. Moreover, in an oligopoly, other firms may respond to any one firm's price changes, so a rational firm will not set its price to maximize profits taking as given its rivals' prices. Indeed, some economists have suggested that this often yields a "kink" in the demand curve facing a firm once rival responses are taken into account. Thus, in estimating the Actual Loss, it is better to give little or no weight to the firm's pre-merger

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[12] The Appendix shows that this approximation is exact (in the way we use it) when demand is linear. Of course, demand will never be exactly linear, but for small changes in price, departures from linearity should not normally cause substantial errors, especially when we correct for curvature or kinks in Section 2 below.

[13] This approach to Critical Loss Analysis was explored by Michael Katz and Carl Shapiro (2003), "Critical Loss: Let's Tell the Whole Story," *Antitrust*, and Daniel O'Brien and Abraham Wickelgren (2003) "A Critical Analysis of Critical Loss Analysis," 71 *Antitrust Law Journal* 161. Arguments against relying on such information were presented by David Scheffman and Joseph Simons, "The State of Critical Loss Analysis: Let's Make Sure We Understand the Whole Story," (2003), *Antitrust Source*, November. Michael Katz and Carl Shapiro (2004), "Further Thoughts on Critical Loss," *Antitrust Source*, March, and Daniel O'Brien and Abraham Wickelgren (2004), "The State of Critical Loss Analysis: Reply to Scheffman and Simons," *Antitrust Source*, March, responded to Scheffman and Simons.

choice of price, and instead to seek direct evidence about demand responsiveness. Such evidence can include econometric or marketing studies, and/or an intuitive evaluation of qualitative facts about what it would take for a customer to substitute away from a group of products. If the answer appears at odds with firms' pre-merger prices and margins, one should not be unduly puzzled or concerned.

In Sections 2 to 4 we address the concerns raised in the second bullet about the use of pre-merger margins to estimate the Actual Loss. To make the concept concrete, however, we begin by reviewing how pre-merger margins can be used in the simplest case.

C. How Pre-Merger Choices Reveal Demand Sensitivity: The Simple Case

KSOW develop a simple test for market definition that uses two ingredients. The first ingredient is the pre-merger margin, *m*. The second is the Aggregate Diversion Ratio, *A*, which is the fraction of the sales lost by one product, when its price alone rises by a SSNIP, that go to other products in the candidate market. Equivalently, one can think of $(1-A)$ as the fraction of a product's demand elasticity that consists of substitution to goods *outside* the candidate market.

Thus the elasticity of demand facing the hypothetical monopolist is only a fraction $(1-A)$ of that facing any one product. This directly implies an estimate of Actual Loss of $L=(1-A)\varepsilon s$; this estimate is precise if demand is linear for small price changes. So, if we can use the Lerner Equation, the Actual Loss equals $L=(1-A)(s/m)$. For example, with a SSNIP of 5%, a margin of 45%, and an Aggregate Diversion Ratio of 25%, the Actual Loss is estimated to be 8.3%, while the Critical Loss is 10%, so the group of products would form a market. Following this logic gives:

Proposition 1 (KSOW): If each firm owns a single product and prices to maximize its profits taking as given all other prices, and if demand for each product is linear in price for small price changes from the pre-merger price, then a symmetric group of products forms a relevant market under break-even analysis if and only if $A \geq \frac{s}{m+s}$.

We do not suggest that this revealed preference approach is the only way to estimate Actual Loss. But nor should it be ignored. If one had a reasonably reliable estimate of *m* and of the Aggregate Diversion Ratio *A*, one should be skeptical of a separate estimate of Actual Loss that departed much from the predicted value of $L=(1-A)(s/m)$. In our numerical example, a separate estimate of Actual Loss much greater than 8.3% would require reconciliation with the estimate of 25% for the Aggregate Diversion Ratio. We discuss how to handle such conflicting evidence in Section 5 below.

Evidence on the Aggregate Diversion Ratio may be gleaned from surveys of customer switching patterns or past customer responses to changes in prices or product availability. Econometric evidence based on demand responses to price changes also can help in measuring *A*. As usual, one must seek to avoid inadvertently measuring price responses to demand shifts. Recall that *A* is calculated on the assumption that the price of one product changes and the other prices do not.

Proposition 1 implies that a seemingly narrow group of products will often form a relevant market according to the *Guidelines*. With the standard SSNIP of $s = .05$ and with a moderate margin of $m = .45$, a group of products forms a relevant market if $A \geq 0.1$. In many intuitively defined "industries," the aggregate diversion ratio would be far higher, so narrower markets may well exist within the industry. For instance, if the price of one model, or brand, of cars were to rise by a SSNIP, quite a few customers would no doubt substitute away—but we would expect that most of them would substitute away to some other car. Thus, if gross margins are about 45%, there would be a relevant product market considerably narrower than "cars." For example, if twenty percent of BMW customers would substitute to Mercedes or Audi following a SSNIP by BMW, and conversely, then the hypothetical monopolist test suggests that "German luxury cars" would be a relevant market.[14] Certainly this arithmetic suggests that "cars" would easily be a relevant market, and the hypothetical monopolist methodology would not imply that pickup trucks or minivans would need to be included.

The *Whole Foods* court appeared to believe that a group of products could not form a market if A is significantly below one-half, but our analysis shows that this view is inconsistent with the *Guidelines* if, as is normally the case, m substantially exceeds s.[15]

But what if the assumptions behind Proposition 1 are not accurate? In Section 2 we generalize Proposition 1 to cover industries in which the pre-merger firms respond to one another's price changes. In Section 3 we generalize Proposition 1 to allow for the possibility that demand is not linear in price—in particular that demand becomes more sensitive to price increases as the price rises above the pre-merger level. In Section 4 we discuss how Critical Loss Analysis can be useful even when firms' real-world profit functions include various subtle and complex factors.[16]

2. Accounting for Pricing Responses by Rivals

Much quantitative merger analysis assumes in practice that each firm sets its price taking as given the other firms' prices. So far, we have followed that approach. However, this may not

[14] As throughout this paper, we focus on whether a SSNIP would be profitable, not on whether the candidate market contains the closest substitutes, in order, starting from one product of one of the merging firms.

[15] The Court stated: "In other words, when Whole Foods enters an area that has a Wild Oats store, its sales do not come overwhelmingly from Wild Oats, but primarily from other stores; the main competitive interaction is between Whole Foods and "other" grocery retailers." *Opinion* in Federal Trade Commission v. Whole Foods Market and Wild Oats Market, District Court for the District of Columbia, August 16, 2007, [p.33, slip opinion]. While the public information on *Whole Foods* does not appear to include an estimate of m, there is evidence that the margin was far larger than the size of a SSNIP. The public version of the rebuttal report of the FTC's expert Dr. Kevin Murphy indicates (paragraph 28) that even with an aggregate diversion ratio of only 0.1 to 0.2, premium/natural organic supermarkets would constitute a relevant market according to a test equivalent to Proposition 1. SSNIP sizes of 5% and 1% were discussed earlier in his report. Using a SSNIP of 5% and an aggregate diversion ratio of 20% would, according to Proposition 1, suggest a gross margin of 20%, four times the SSNIP. Using a SSNIP of 1% and an aggregate diversion ratio of 10% would suggest a gross margin of 9%, nine times the SSNIP.

[16] In a few places we combine these analyses, but that is largely left for future work: we primarily relax the simplifying assumptions one at a time.

accurately reflect firms' actual behavior in many oligopolies in which mergers would be challenged these days. Indeed, whether a firm tracks, and responds to, other firms' price changes is often taken as an indicator of whether they compete or are in the same market.[17]

If pre-merger oligopolists are in the habit of responding to one another's price changes, then a firm that contemplates a price change will not care how its sales *would* respond if other prices were held fixed, but will seek to predict how its sales *will* respond once others react to that change as they are expected to do in the real-world oligopoly.[18] In general, then, revealed-preference information illuminates price sensitivity on this *residual demand curve*, not (directly) on the conventional demand curve that holds other prices fixed.[19]

Again, some fraction of the sales lost by any one product through its SSNIP go to other products in the candidate market, but now this fraction must be measured using the residual demand curve, i.e., accounting for the price responses of the other products. We call this the Residual Aggregate Diversion Ratio, or A^*. The Appendix proves:[20]

Proposition 2: If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, and if demand for each product is linear in price for small price changes starting from the pre-merger price, then a symmetric group of products forms a relevant market under break-even analysis if $A^* \geq \frac{s}{m+s}$, but may do so even if that condition fails to hold.

In the usual case where firms expect accommodating responses, meaning that rivals at least partly match price changes, less of each firm's lost sales will remain within the candidate market than would if those rivals did not respond. Thus $A^* < A$, so using A and Proposition 1 could incorrectly suggest that a group of products is a relevant market. Intuitively, if pre-merger firms

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[17] For example, the *Whole Foods* court noted that Whole Foods tracks prices in mainstream supermarkets. *Opinion*, p.49-50.

[18] Economists will recognize this approach as involving "conjectural variations," as developed by A. Bowley, (1924), *Mathematical Foundations of Economics*, New York, Oxford University Press and extended by Timothy Bresnahan, (1981), "Duopoly Models with Consistent Conjectures," *American Economic Review,* 71:934-945 and Robert Porter, (1982), "Oligopoly and Consistent Conjectural Variations," *Bell Journal of Economics*, 13:197-205. The theory of conjectural variations has been criticized in economic theory, because it does not lay out from first principles a game-theoretic model of oligopoly. This is something of a bum rap. Economic theory does not suggest that oligopolies will tend to be characterized by static Nash competition; it is just that game theorists understand that case best. But the fact is that oligopolists often do respond to rivals' price changes, so a theory that recognizes that fact has some considerable advantage over a more elegant theory that denies it.

[19] The importance of the residual demand curve for assessing market power is not new in antitrust. See, especially, Jonathan Baker and Timothy Bresnahan, (1988) "Estimating the Residual Demand Curve Facing a Single Firm," *International Journal of Industrial Organization* 6:283–300.

[20] The Appendix explains the technical assumptions underlying Proposition 3. We assume that the products are "strategic complements," so when one firm raises its price it expects the others to raise their prices (or leave them unchanged). We also assume that the prices set by firms in the candidate market are not materially influenced by any responses they anticipate from firms *outside* the candidate market. Our results would need to be modified if this condition is not met.

face substantial accommodating responses, then they are not competing very strongly, so there is less of a change from that situation to that of control by the hypothetical monopolist, who therefore might find a SSNIP unprofitable.[21] Proposition 2 offers a conservative check on that possibility, by giving a sufficient (but not necessary) condition using the smaller value A^*.

The Residual Aggregate Diversion Ratio, A^*, may be estimated from industry experience of responses to price changes. Econometrically, it would be important to ensure that those price changes are indeed initiated by the one firm, perhaps following changes in its firm-specific costs, or changes in managerial assessment of market conditions, as opposed to general price shifts prompted by industry-wide cost shocks or by shifts in demand. Evidence on A^* could also come from documents of the merging parties, especially those analyzing the profitability of a price change by estimating the extent to which sales will *in fact* (in the pre-merger oligopoly) be lost to (or taken from) various rival products. For example, a merging party might have documents predicting rival pricing responses. One can estimate A^* by combining these expected pricing responses with survey data indicating customer switching patterns in response to price changes. Furthermore, evidence regarding A will bear on A^*, either through the inequality $A^* \leq A$ or to the extent that one might be able to trace through the differences between the two.[22]

Estimation of A^* will often be more reliable than estimation of L, because A^* is directly relevant to real-world pre-merger pricing decisions faced by the firms: "If we raise our price, and our rivals respond as we expect, how many customers will we lose and what alternatives will they pick?" In contrast, L inherently involves a hypothetical that is typically a significant, artificial departure from the real, pre-merger world: "If we and a collection of our rivals raise our prices in unison, how many customers will we lose?" This question is unlikely to be contemplated by suppliers in the normal course of business (unless they are engaged in price fixing or the industry dynamics involve very strong price-matching responses).

3. Demand More Sensitive to Price Increases than Price Decreases

Scheffman and Simons (2003, p.6) criticize the KSOW test in Proposition 1 because it assumes that customers are equally sensitive to price increases and price decreases: "But the price

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[21] The *Guidelines* recognize that pre-merger coordination may not persist, so the "but-for merger" price from which the SSNIP should be measured may be lower than the pre-merger price. Section 1.1 in the *Guidelines* states: "…the Agency will use prevailing prices of the products of the merging firms and possible substitutes for such products, unless premerger circumstances are strongly suggestive of coordinated interaction, in which case the Agency will use a price more reflective of the competitive price." With strongly accommodating pre-merger behavior, it may not be appropriate under the *Guidelines* to measure the SSNIP starting from pre-merger prices, and the test in Proposition 3 would need to be modified. Our approach offers a possible way to do so—using the no-reactions Diversion Ratio—while as far as we are aware other quantitative techniques do not. If one believes that pre-merger pricing reflects (perhaps tacit) coordination that ought not to be taken as the but-for world, market definition might begin from a lower price, just as it ought in circumstances threatened by the Cellophane Fallacy.

[22] The relationship between A and A^* depends upon the degree to which rivals match a single firm's pricing changes and the impact of those reactions on the firm's sales (as measured by diversion ratios). The proof of Proposition 3 in the Appendix derives a formula for the relationship between A and A^*.

elasticity might be significantly different for price increases than for price decreases." We now show how Proposition 1 can be modified to address this possibility.

Suppose that a single product loses sales in response to a price increase, accounting for responses by other firms in the candidate market, at a rate $1+k$ times the rate at which it gains sales in response to a tiny price decrease.[23] With this definition, which is simply an arithmetical way to keep track of such possible asymmetry on the residual demand curve, Proposition 2 becomes:

Proposition 3: If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, a symmetric group of products forms a relevant market under break-even analysis if and only if

$A^* \geq \dfrac{\frac{s}{m+s}+k}{1+k}$ **which is equivalent to** $k \leq \dfrac{A^*-\frac{s}{m+s}}{1-A}$**.**

While it is possible that customers are much more sensitive to price increases than price decreases, with many heterogeneous customers it would seem rather unusual for this effect to be significant, once all of these customers are aggregated. Hence, we start with some skepticism about the empirical likelihood of k being much greater than zero based on asymmetric customer responses. Following Katz and Shapiro (2003), it seems reasonable to require a party arguing for $k > 0$ based on asymmetric customer responses to present evidence in support of this claim.

Another possibility is that customers are equally sensitive to price increases and decreases but *rivals* have asymmetric responses. Suppose, in particular, that rivals more fully match price cuts than they do price increases: this is the "kinked demand curve" theory of oligopoly.[24] The Appendix shows how such behavior can imply $k > 0$ and provides a method of calculating k.[25]

How would one learn about the demand asymmetry parameter, k? One approach is simply to treat it as part of estimating demand econometrically. Another approach would be to study the rate at which firms pass through changes in their marginal costs.[26] The Appendix derives an approximate relationship between that pass-through rate, r, and k:

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[23] The Appendix gives a technical definition of k. If the demand curve is not literally "kinked," but only curved, then it is not necessary to specify that the tiny price change is a decrease.

[24] See Paul Sweezy (1939), "Demand Under Conditions of Oligopoly," *Journal of Political Economy*, 47:4, 568-573. Sweezy refers to the "imagined demand curve," which we now know as the "residual demand curve." See also R. I. Hall and C.J. Hitch (1939), "Price Theory and Business Behavior," *Oxford Economic Papers*, 2:12-45.

[25] If a candidate market includes all of the rival products whose asymmetric responses kink the residual demand curve for any one product, then as Katz and Shapiro (2004) observe, there is no kink in the demand curve facing the hypothetical monopolist.

[26] This is the single-firm pre-merger pass-through rate. Our companion paper, "Cannibalization, Pass-Through, and Market Definition," extensively discusses the use of pass-through rates for market definition. Pass-through rates commonly arise in merger analysis when the impact of merger efficiencies on consumers is studied. See, for example, Luke Froeb, Steven Tschantz, and Gregory Werden, (2005), "Pass-Through Rates and the Price Effects of Mergers," *International Journal of Industrial Organization*, 23:703-715.

$$k \approx \frac{s}{m}\frac{1-2r}{2r} \text{ or equivalently } r \approx \frac{1}{2}\frac{s}{s+mk}.$$

Intuitively, if demand is more sensitive to price increases than to price decreases, a firm will be reluctant either to pass through cost increases (because demand will respond sharply) or to pass through cost decreases (because demand will not respond sharply), so that price will tend to be "sticky." It is important here to distinguish a low pass-through rate of cost changes into profit-maximizing prices versus the possibility that prices are sticky in the short term simply because it may be costly to change price and, with high margins, a firm's profit function is not very sensitive to price.

We are interested in the case where demand is more sensitive to price increases than price decreases, i.e., $k > 0$. This corresponds to a pass-through rate of $r < 1/2$.[27] To illustrate using the numbers from above, if the SSNIP is 5%, the margin is 45%, and the pass-through rate is 25%, then the asymmetry parameter is $k = 0.11$.

If *k* can be estimated, either directly or based on the pass-through rate, the first inequality in Proposition 3 implies a critical value of the Residual Aggregate Diversion Ratio such that the candidate products form a relevant market. With the numbers just given, Proposition 3 states that the products form a relevant market if and only if $A^* \geq .19$. This inequality can be compared with the inequality $A^* \geq .10$ that applied with these same parameters under conditions of symmetric demand response, i.e., $k = 0$, which is consistent with a 50% pass-through rate.

Alternatively the second inequality in Proposition 2 can be used to calculate a critical value of the asymmetry parameter, $k_{\max}$, such that the products under study form a market if and only if $k \leq k_{\max}$. Using these same parameters, $s/(m+s) = 0.1$, so $k_{\max} = (A^* - 0.1)/(1 - A^*)$. If $A^* = 0.2$, then $k_{\max} = .125$, which corresponds to a critical value for the pass-through rate of about 25%.

One procedure for incorporating these issues would run as follows. Begin with a default assumption that demand is equally sensitive to price increases and price decreases: $k = 0$.[28] Under this default assumption, Proposition 2 tells us that the symmetric group of products forms a market if and only if $A^* \geq s/(m+s)$. If good evidence on A^* implies that this condition holds, there would be a presumption that these products form a relevant market, but this presumption could be rebutted.

One way to rebut the presumption would be to produce other evidence on Actual Loss. If that other evidence suggests that $L > s/(m+s)$, it would conflict with the evidence suggesting that

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[27] Technically, if $r > 1/2$, then $k < 0$ and the condition in Proposition 3 remains sufficient.

[28] Alternatively, one might begin with a different default assumption or presumption about *k*, based on empirical evidence not specific to the case at hand. Although we would not want to over-weight evidence consisting of economic theorists' and econometricians' conventional choices of functional forms for study, we note that while linear demand has $k = 0$, constant-elasticity and several other widely used "standard" demand systems have $k < 0$.

$A^* > s/(m+s)$. In Section 5 we offer some comments on how a court might weigh such conflicting evidence, but our principal point is that it should do so, and should not ignore the evidence on Actual Loss that is based on estimating A^*.

Another way to rebut the presumption would be through evidence that demand is sufficiently more sensitive to price increases than to price decreases. The necessary level of asymmetry k_{max} is given by Proposition 3. One might use a sliding scale of rebuttable presumption, with more convincing evidence required for larger k_{max}.

This approach treats evidence concerning demand response (such as company documents, customer interviews, marketing surveys or econometric estimates of demand) somewhat differently than does Critical Loss analysis as it is often currently performed. In many cases, such as *Whole Foods*, an expert for the merging parties estimates Actual Loss using evidence that says little or nothing about *asymmetric* demand response, neither explicitly estimating k nor calculating k_{max}, but that tends to suggest that the demand facing the hypothetical monopolist is sensitive to price.[29] Combining such evidence with a calculation that, with fairly high margins, Critical Loss is not very large, some experts and courts have inferred that a group of products is too narrow to be a relevant market. But this approach ignores pre-merger pricing information: a high demand elasticity would also suggest that firms were pricing irrationally high prior to the merger. Thus it puts little or no weight on firms' own pre-merger pricing decisions, or is very willing to believe a high value of k, or both. This approach, in contrast, discounts evidence that would suggest high demand elasticity if it conflicts with the evidence from firms' pre-merger pricing decisions, unless that conflict is resolved by evidence on demand response or industry dynamics.

4. Complexities in Real-World Profits

Real-world firms often do not maximize the direct, readily quantifiable profits from any one product considered in isolation. When a firm sells competing or complementary products, it will naturally account for spillover effects on profits when setting the price of any one product. Plus, dynamic and intangible considerations such as customer loyalty, reputation, network effects, and learning curves commonly arise.

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[29] Criticizing the economic expert for the merging parties, the economic expert for the FTC stated: "Economic theory makes no prediction that consumers will respond more to price increases than to price decreases, and Dr. Scheffman provides zero evidence that such asymmetric responses would be expected in this case, or any other one. All of the qualitative evidence he relies on is equally supportive of large responses to price increases and decreases, both for existing firms and for a hypothetical PNOS monopolist." Public Version of the Expert Report of Dr. Kevin M. Murphy, Ph.D., ¶14. A different way in which direct estimates of Actual Loss sometimes fail to confront contrary evidence from revealed preference is if the direct estimate merely recites a number of ways by which demand elasticity *could* arise—ways in which customers *could* substitute—but does not reconcile this estimate with the evidence from pre-merger margins that those channels of substitution do not *in fact* cause high pre-merger elasticity.

When such factors are significant, it matters whether or not one views the hypothetical monopolist as inheriting pre-merger firms' concern for those factors. If so—an assumption that might place more strain on our assumption of symmetry[30]—then we can extend the analysis above relatively simply. If not, then there are reasons to be pessimistic about the likely contribution of market definition to a reasonable presumption or screen concerning a merger's competitive effects.

First, then, suppose that the various factors that enter into the pre-merger firms' profit functions are also included in the hypothetical monopolist's profit function. In the symmetric case, this seems the most useful and appropriate way to interpret the hypothetical monopolist test.[31] After all, the point of the test is to identify the collection of firms that would find it profitable to impose a SSNIP, so that changes in concentration among that group of firms tell one something about the likely competitive effects of the merger. Concern for customer reputation or for follow-on sales of a tangible complement applies before the proposed merger and will apply afterwards. Therefore, it seems natural to include it in any analysis intended to be informative about competitive effects.[32]

The calculations above did not take such factors into account, either in the simplified profit function used for the hypothetical monopolist or for the profit functions of the pre-merger firms. We now show how to use pre-merger pricing information in markets where these complexities are important.

To illustrate, suppose that the pre-merger firms all sell a complementary product as well as the primary product being studied. Suppose that each firm expects some profitable sales of the complement to follow each sale of the primary product. For example, in a candidate market for enterprise computers, each manufacturer may anticipate profitable follow-on sales of information technology management services to the buyers of its computers. Then a pre-merger firm sets the price of its primary product to maximize not simply the direct profit from sales of the primary product, $(p-c)x$, but rather $(p-c)x+Bx$, where B represents the anticipated follow-on benefits (profits) from selling one more unit of the primary product.[33] This formulation is general

[30] Of course, these complexities may differ for the different firms in the market. As noted above, in practice Critical Loss Analysis is performed by considering a uniform SSNIP applied to symmetric products. We retain the assumption of symmetry in this section and refer readers to Moresi et al. (op. cit.) and to our companion paper, "Cannibalization, Pass-Through, and Market Definition," for analysis of asymmetric situations.

[31] The *Guidelines* do not explicitly state what factors should be included in the profit function of the hypothetical monopolist, and we are not aware of this issue being addressed in other published work or Agency guidance.

[32] Scheffman and Simons (2003) emphasize that the Critical Loss is "just arithmetic." That is only true if one uses the highly simplified profit function for the hypothetical monopolist. While using that profit function is reasonable if the excluded factors are small, when they are large it introduces a significant change in incentives having nothing to do with the hypothesized end to competition among the products in the candidate market. It thus muddies the waters of market definition. We show below how using the simplified profit function can lead to absurd results.

[33] We continue to assume that the firm maximizes its overall profits. The critical-loss methodology, and indeed almost all modern antitrust economics, is based on evaluating firms' profit incentives (and the *Guidelines* follow this approach regarding the hypothetical monopolist). Scheffman and Simons (2003) observe that firms do not always exactly maximize profits. This is surely true, but by itself tells us nothing about whether merger policy should be

enough to handle a wide range of complexities, including network effects, learning-by-doing, and reputation effects as well as multi-product firms. An outside observer may or may not be able to estimate accurately the benefit B in a particular case, but even when it is difficult to do so, it may at least be feasible to determine whether B is positive or negative. We believe that most of the intangibles that businesses typically focus on involve a positive value of B, so we assume here that $B \geq 0$. That is, if anything, businesses typically like to make sales even past the point where a simple quantitative profit analysis would recommend.

Introducing additional per-unit benefits of B is economically equivalent to reducing the net cost of selling a unit from c to $c - B$. The true economic margin is thus $m^* = (p - c + B)/p$ rather than $m = (p-c)/p$, the accounting margin. We write $m^* = m + b$ where $b = B/p$. Replacing the accounting margin m with the economic margin m^*, all of the analysis leading up to Propositions 1, 2, and 3 can be repeated, but now the presence of these follow-on (perhaps intangible) benefits does not undermine the Lerner equation, which takes them into account. Thus Propositions 1, 2, and 3 remain valid, replacing m with $m^* = m + b$. For example, the condition in Proposition 2 becomes $A^* \geq \frac{s}{m^* + s} = \frac{s}{m + b + s}$.

If one can reliably quantify b then one can use these formulae directly to diagnose relevant markets in this more complex environment. In other cases, it will not be possible to quantify b and thus calculate m^*. However, if $B > 0$ then $m^* > m$, so if the conditions provided in Propositions 1, 2, and 3 hold using the accounting margin m, then they hold using m^*, so one can all the more strongly conclude that the candidate group of products form a relevant market.

Proposition 4: If each firm receives some additional benefit from selling its primary product, and if these benefits are not included in the accounting margin *m*, then if the conditions in Propositions 1, 2, and 3 hold using that accounting margin, the candidate group of products must form a relevant market under break-even analysis, so long as these benefits also are included in the profits of the hypothetical monopolist.

We now address the possibility that $B < 0$, i.e., that each firm incurs some cost, not measured in the accounting margin, when it sells one more unit of the primary product. This possibility arises if the primary product cannibalizes sales from substitute products owned by each firm that are not in the candidate market.[34] In this situation, the tests in Propositions 1, 2, and 3 can incorrectly report that a candidate group of products is a relevant market: the true economic margin is lower than the accounting margin, so the elasticity of demand for each product is higher than would be inferred from the Lerner Equation using the accounting margin, leading to a larger Actual Loss. One way to analyze that situation is to perform Critical Loss Analysis on a

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more or less restrictive, and is not something to be used piecemeal to object to one inference from profit incentives in an analytical structure replete with them.

[34] This case has recently been studied by Serge Moresi, Steven Salop, and John Woodbury in "Implementing the Hypothetical Monopolist SSNIP Test with Multi-Product Firms," December 2007.

broader group of products including those substitute products.[35] Alternatively, if the cannibalization effect can be measured, one can use the (lower) true economic margin instead of the accounting margin in Propositions 1, 2, and 3.

While we think it makes sense to include B in principle in the hypothetical monopolist's profit function in the symmetric case, one might read the *Guidelines* not to do so. Surprisingly, the issue does not seem to have attracted explicit discussion, and we do not take a strong general position here, but simply observe that omitting a substantial B can be expected to lead to market definitions that may be uninformative or misleading.

The fundamental idea of market definition for merger analysis is to provide a simple preliminary gauge of the extent to which a merger's removal of competition between the merging firms' products will threaten competition. To do so, it asks how important that competition is in restraining (for instance) prices, pre-merger. It seems strikingly uninformative to observe, for instance, that if, instead of removing that competition, one removed each product's link to an important complement (that is present pre-merger and will be present post-merger), prices would rise substantially. But this is all the hypothetical monopolist test would be telling us if we stripped the hypothetical monopolist of pre-merger firms' significant concerns for their complements: that is, if $b > s$, one would find single-product markets.[36]

5. Conflicting Evidence

Because market definition is often central in antitrust litigation, courts and agencies are often confronted with conflicting Critical Loss Analyses. We offer some suggestions for using revealed preference analysis to help confront such conflicting economic evidence.

Suppose that a SSNIP of 5% will be used and that there is no dispute that the accounting margin is 45%.[37] If there are no significant intangible benefits, these values for m and s imply that the Critical Loss is 10%. Suppose, however, that the opposing experts disagree over the Actual Loss

[35] Strictly speaking, doing this might not adhere to the algorithm in the *Guidelines* for adding products in the order of "next-closest substitutes," and "generally" stopping with the smallest group of products that forms a market. In practice, this algorithm is rarely followed slavishly, in part because the information necessary to do so is typically unavailable.

[36] In general, with $B > 0$, the Appendix shows that the hypothetical monopolist test systematically leads to *narrower* markets if the hypothetical monopolist's profits do not include B than if they do.

[37] Since m is important in Critical Loss Analysis, one might expect it to be controversial, as it typically is in predatory pricing cases. If Actual Loss is estimated separately, then estimates of m are used only in calculating Critical Loss, where a higher estimate of m lowers estimated Critical Loss and thus tends to lead to broader markets. Typically one would then expect to see the government arguing for lower m and the merging firms for higher m, although it is not always the case that broader markets assuage competitive concerns. If, on the other hand, Actual Loss is estimated from revealed preference, then a higher estimated m reduces the estimates of both Critical Loss and Actual Loss; as it turns out (see Propositions 1 to 3) a higher estimate of m tends on balance to lead to narrower markets.

to be expected if a hypothetical monopolist over the government's proposed market were to impose a SSNIP.

Professor A, the government's economic expert, performs a study finding that the Actual Loss would be only 8.3%. Because this is less than the Critical Loss of 10%, she finds that the government's proposed market is indeed a relevant market.

Professor B, the expert for the merging parties, performs a different study concluding that the Actual Loss would be $L = 15\%$. Because this exceeds the Critical Loss of 10%, he testifies that the government's proposed market is not a relevant market.

Can we help the judge assess the reliability of the conflicting estimates?

If neither study used revealed-preference methods, one can ask what each would imply for the Residual Aggregate Diversion Ratio, *A**. To do so, we ask what loss of sales an individual product would experience if it unilaterally raised its price by the 5% SSNIP. Given the margin of 45%, the Lerner equation suggests an elasticity of residual demand of $1/.45$, or 2.2. With linear demand, the loss of sales from a 5% price increase would be 2.2 times 5%, or 11%. This is less than the 15% Actual Loss estimated by Professor B, the defense expert. How can the Actual Loss for the hypothetical monopolist be *more* than the loss for a single product?

As Section 3 discussed, Professor B could defend his $L = 15\%$ estimate of Actual Loss by arguing that demand is more sensitive to price increases than price decreases. With such demand asymmetry, the Lerner equation for pre-merger firms gives an estimate of Actual Loss for the hypothetical monopolist of $L = (1+k)(1-A^*)(s/m)$. Since $s/m = 0.11$, Professor B's estimate that $L = 0.15$ implies that $(1+k)(1-A^*) = \frac{0.15}{0.11} = 1.36$, or $k = \frac{1.36}{1-A^*} - 1$, so $k > .36$ (so long as *A** is positive). This value of *k* corresponds to a pass-through rate of no more than 22%. While *k* might be this large, as discussed in Section 3, we would recommend further probing of Professor B's Actual Loss estimate, since it implies such a sharply curved (or kinked) residual demand curve. The Court could ask for evidence that the pass-through rate is less than 22%.

Defense expert Professor B might alternatively point out that many factors other than accounting profits enter into firms' pre-merger pricing decisions. Our analysis in Section 4 implies that this is not a good answer if $b > 0$, as we expect will normally be the case. For then the true economic margin exceeds the accounting margin of 45%, so the Lerner equation indicates that the elasticity of demand facing any given product is *lower* than the previous estimate of 2.2. For example, if selling one more unit generates intangible benefits valued at 5% of the price, so $b = .05$, then the economic margin is $m^* = m + b = .45 + .05 = 50\%$, so the Lerner equation implies that the elasticity of demand facing any one product is 2.0. Therefore, the loss of sales for a single product following a unilateral SSNIP will be 10%, which casts even more doubt on Professor B's Actual Loss estimate of $L = 15\%$. Thus a challenge to the use of the Lerner Equation here must involve either a claim that pre-merger firms were pricing *above* their profit-maximizing level (and our use of residual demand implies that this would not be simply a result of oligopoly interdependence), or face important intangibles discouraging additional sales, or else a radical challenge to the use of the profit maximization methodology.

With the numbers given above, government expert Professor A's estimate that $L = 0.83$ is easily calculated to be consistent with an aggregate diversion ratio of 25% if *k* is zero. Specifically, recall that the Lerner equation applied to a 45% gross margin implies a product-level residual demand elasticity of 2.2. If $A^* = 0.25$ then the predicted demand elasticity for the hypothetical monopolist is $(1-0.25)\times 2.2 = 1.65$, so that a 5% SSNIP would lead to an Actual Loss of $1.65\times 0.05 = .083$ or 8.3%. The Court could ask whether this 25% estimate for *A** is consistent with the evidence.[38]

Another possibility is that one of the experts relied on the revealed-preference method. Suppose for instance that the government's expert, Professor A, put less weight on marketing and econometric studies of demand but conducted a study of demand substitution patterns in response to price changes initiated by one product. Her study yielded an estimate $A^* = 0.25$, and she testified that this was her primary basis for her $L = 8.3\%$ estimate. In this case, obviously, consistency with *A** is not an issue. Now the two experts have arrived at their conflicting estimates of *L* via more fundamentally differing paths. Professor A estimated *A**, inferred *L*, and found that the inferred estimate of *L* was less than the Critical Loss. Professor B estimated *L* to be greater than the Critical Loss. They embody the two bullet points in Section 1.B.

In this situation, we hope the Court would recognize and confront the tension between the two approaches. Naturally, the Court will want to assess the quality of the evidence underlying the estimate of $A^* = 25\%$ by the government's expert and the estimate of the Actual Loss of $L = 15\%$ by the expert for the merging parties.

The Court may be tempted to give greater weight to the separate "direct" estimate of *L* than to the indirect estimate of Actual Loss based on an estimate of *A** combined with use of the Lerner Equation. However, we believe such an approach would not be justified, for three reasons. First, the version of the Lerner Equation developed in this paper is very general and flexible, allowing for oligopolistic interactions, for curvature of demand, and for many other factors to enter into firms' pricing decisions. Rejecting this version of the Lerner Equation is tantamount to rejecting the whole profit-maximization methodology that underlies the *Guidelines* and indeed much if not all of antitrust economics as currently practiced. Second, as noted above, it may well be possible to obtain a more reliable estimate of *A** than of *L* based on documentary evidence, because *A** relates more closely to questions that the suppliers actually face in the real, pre-merger world. Third, since Critical Loss Analysis involves comparing two estimated quantities (the Critical and Actual Losses), its accuracy depends in potentially complex ways on the accuracy of the two estimates. In particular, if a method errs in the same direction in its estimation of each quantity, it may be more robust and accurate than another method whose errors are less correlated. Further work on this issue could prove helpful.

[38] The Court also can ask what value of *k* would bring the estimate of *A** in line with the other evidence.

6. Break-Even and Profit-Maximizing Tests

As we noted in the previous paragraph, the assumption of profit maximization permeates much, if not all, of antitrust economics as currently practiced. The break-even version of Critical Loss Analysis we have explored so far does not analyze what a hypothetical monopolist would find *most* profitable, but only asks which of two pricing patterns it would find *more* profitable. In doing so, it departs not only from usual economic and antitrust methodology, but also from the *Guidelines*. Such a departure is somewhat built-in to an approach of separate estimation of Critical and Actual Losses, but is avoidable if one is willing to take as a working approximation that demand is roughly linear, or to use another functional form for demand. Thus we can modify our Propositions 1 through 4 to work with the profit-maximizing version of the SSNIP test called for in the *Guidelines*.

A. Relationship Between the Two Tests

The Appendix provides mild conditions such that if the profit-maximizing response to hypothetical monopolization is to impose at least a SSNIP, then a SSNIP is more profitable than the status quo.[39] Intuitively, if a SSNIP is a partial move toward profit maximization (from the status quo) then it will likely yield some increase in profit. However, even when these conditions are met, the converse need not hold, and no reasonable conditions suggest themselves under which one could make that inference. Thus, for a given size of SSNIP, the break-even test is easier to satisfy, and will tend to suggest narrower markets. See subsection B below.

Further work is needed to learn more about which of the two tests is more robust, i.e., less sensitive to errors in the measurement of the Actual Loss, the Residual Aggregate Diversion Ratio, the curvature of the residual demand curve, and the pre-merger margins.

B. Linear Demand: Modifying Propositions 1 and 2

Recall that Propositions 1 and 2 assumed linear (residual) demand. The Appendix shows that for a firm—including a hypothetical monopolist—with constant marginal cost and linear (residual) demand, the profit-maximizing price change from any status quo is just half the break-even change. Thus, as Katz and Shapiro (2003) noted, it is profit-maximizing to impose (at least) a SSNIP s if and only if a price increase of $2s$ is more profitable than the status quo. As a result, we can readily modify Propositions 1 and 2 as follows:

[39] These conditions will often be met, but not always. Discussing the *Sungard* case, Katz (2002) notes that they would *not* be met if a substantial number of customers would switch in response to a 5% SSNIP, making it unprofitable, but few additional customers would switch in response to a considerably larger price increase, making the latter profit-maximizing. He also suggests that even if *in fact* a 5% SSNIP would be profitable, it might be easier to *prove* that a larger price increase would be profitable.

Proposition 1A (KSOW): If each firm owns a single product and prices to maximize its profits taking as given all other prices, and if demand for each product is linear in price for small price changes from the pre-merger price, then a symmetric group of products forms a relevant market under profit-maximizing analysis if and only if $A > \frac{2s}{m+2s}$.

Proposition 2A: If each firm owns a single product and prices to maximize its profits accounting for price responses by other firms in the candidate market, and if demand for each product is linear in price for small price changes starting from the pre-merger price, then a symmetric group of products forms a relevant market under profit-maximizing analysis if $A^* \geq \frac{2s}{m+2s}$, but may do so even if that condition fails to hold.

The logic underlying Proposition 4 carries over to the profit-maximizing version of the SSNIP test, so we have

Proposition 4A: If each firm receives some additional benefit from selling its primary product, and if these benefits are not included in the accounting margin *m*, then if the conditions in Propositions 1A and 2A hold using that accounting margin, the candidate group of products must form a relevant market under profit-maximizing analysis, so long as these benefits also are included in the profits of the hypothetical monopolist.

C. Demand More Sensitive to Price Increases: Modifying Proposition 3

If demand is more sensitive to price increases than price decreases, the profit-maximizing analysis is significantly more complex than the break-even analysis. The profit-maximizing price increase depends upon the precise shape of the demand curve at prices above the pre-merger price. The rate at which cost changes are passed through into price changes features prominently in this analysis. Our companion paper offers several new and powerful tests for market definition with non-linear demand using the profit-maximizing SSNIP test in the *Guidelines.*[40]

7. Conclusion

We have developed new and improved tools for using pre-merger pricing information in Critical Loss Analysis to help define relevant markets under the *Horizontal Merger Guidelines*. Our overall message is that the complexities sometimes thought to undermine inferences based on pre-merger margins need not do so. We have explored three such complexities: various modes of oligopolistic interaction; greater sensitivity of demand to price increases vs. price decreases; and various complementarities and business intangibles that enter into firms' profits. We have provided two versions of our results: the analysis that applies to the breakeven version of Critical

[40] Joseph Farrell and Carl Shapiro, "Cannibalization, Pass-Through, and Market Definition," op. cit.

Loss Analysis that is used regularly in merger litigation, and one that applies to the profit-maximizing version of the SSNIP test actually called for in the *Guidelines*.

In Section 2, we extended the analysis to cases in which it is important to account for the possibility that oligopolists respond to one another's price initiatives. Katz and Shapiro (2003) discussed this but we take the analysis further. Proposition 2 shows that the natural generalization of Propositions 1 to dynamic oligopolistic conduct continues to provide sufficient conditions for a group of products to form a relevant market.

In Section 3, we extended Proposition 2 to allow for the possibility that residual demand is more sensitive to price increases than to price decreases. We calculated how much asymmetry in the price sensitivity of residual demand must be present for a group of products *not* to form a relevant market. This result, Proposition 3, relies only on Critical Loss arithmetic and the standard assumption that the pre-merger price of each product in the candidate market was set to maximize the profits from that product. So far as we are aware, no such general condition has previously been available. We suggest that the agencies and the courts should require convincing evidence before accepting claims that residual demand is much more sensitive to price increases than to price decreases. We thus believe that if an expert estimates an Actual Loss for a hypothetical monopolist that would imply that a pre-merger firms were starkly failing to maximize profits pre-merger, that fact should be explicitly acknowledged and some skepticism applied. Unfortunately, this does not appear to have happened in *Whole Foods*.

In Section 4, we asked how different approaches fare when business considerations omitted from the simplified profit function substantially affect pre-merger (and likely post-merger) pricing. We showed that, when intangibles such as customer goodwill favor low prices, as we would typically expect, the conditions in Propositions 1, 2, and 3, calculated using accounting margins, remain sufficient for a candidate group of products to form a relevant market so long as the profits of the hypothetical monopolist also include these same intangible factors. This is a major generalization of previous results.

Larger pre-merger margins imply a lower Critical Loss but also, by revealed preference, strongly suggest a lower value of Actual Loss. In that approach, larger pre-merger margins on balance suggest narrower relevant markets, in contrast to what is suggested if one estimates Actual Loss separately. This lesson can be seen in Propositions 1, 2, and 3, which show that a smaller degree of competition among the products in a candidate market (as measured by the Aggregate Diversion Ratio) is sufficient for these products to form a relevant market if pre-merger margin is larger. Intuitively, larger pre-merger margins magnify the cannibalization effect, making it more profitable for the hypothetical monopolist to impose a SSNIP. Another lesson is that relevant markets are narrower, the smaller is the SSNIP used.

Some of our analysis is fairly simple, some not. But where it gets complicated, it does so because it is dealing with complicated reality. One can undertake Critical Loss Analysis in a seemingly simpler manner, but only by ignoring key revealed-preference information and stripping the hypothetical monopolist of key business concerns shared by pre- and post-merger actual firms, thus depleting the market definition exercise of both reliability and relevance.

Even with the new and improved tests developed here and in our companion paper,[41] the whole market definition exercise has some serious drawbacks. We argue in a forthcoming paper[42] that an alternative approach can *replace* the market definition methodology in cases where the primary concern is unilateral anti-competitive effects. We argue that this alternative approach offers a simple and intuitive formulation of unilateral effects that is far more transparent and robust than traditional merger simulation and encourages the integration of marginal-cost efficiencies at an early stage in the analysis.

[41] Joseph Farrell and Carl Shapiro, "Cannibalization, Pass-Through, and Market Definition," op. cit.

[42] Joseph Farrell and Carl Shapiro, "Mergers with Unilateral Effects: A Simpler and More Accurate Alternative to Market Definition," forthcoming. This paper will be available at Shapiro's web site, http://faculty.haas.berkeley.edu/shapiro/.

Appendix

Critical Loss Calculation

If the hypothetical monopolist would lose sales of Z units, then the SSNIP increases the (simplified) profits of the hypothetical monopolist if and only if $(p+sp-c)(X-Z)>(p-c)X$, i.e., if the profits earned on the remaining sales at the higher price exceed those earned on the initial sales at the lower price. This expression can be rewritten as $\frac{Z}{X}<\frac{s}{m+s}$.

Proof of Proposition 1

Let the (point) elasticity of demand facing a single product be given by ε. By the Lerner Equation, $\varepsilon = 1/m$. If a single firm raises the price of its product by a SSNIP, and if the demand for its product is linear over this range in prices, the percentage decrease in its sales will be equal to the percentage increase in price, *s*, times the elasticity, ε. Therefore, the percentage loss its sales will be εs, which equals s/m.

Now consider the hypothetical monopolist that imposes a uniform SSNIP on all of the products in the candidate market. The hypothetical monopolist will recapture a fraction *A* of the sales lost by any one product when its price is raised, since those lost sales will be diverted to products owned by the hypothetical monopolist. Therefore, Actual Loss for the hypothetical monopolist, measured as a percentage of initial sales, will be $(1-A)\varepsilon s$ which equals $(1-A)s/m$. The products form a relevant market if and only if this expression is less than the Critical Loss of $s/(m+s)$. This simplifies to $A \geq s/(m+s)$, proving Proposition 1.

This proof relies on the assumption of linear demand. We used linear demand to conclude that for the single firm's loss in sales will be εs, an expression that is linear in the price increase. If demand is convex, the loss of sales will be smaller and the condition in Proposition 1 will be sufficient but not necessary for the products to form a relevant market. Concave demand is studied in Proposition 2.

The proof also relied on the assumption that the Aggregate Diversion Ratio is constant over the relevant range of prices (between pre-merger prices and the prices after the SSNIP). We used this assumption when we stated that the Actual Loss for the hypothetical monopolist imposing a uniform SSNIP will be $(1-A)\varepsilon s$. One can think of the hypothetical monopolist as imposing the SSNIP on one product after another *seriatim*. The resulting recapture rate will be *A* if the Aggregate Diversion Ratio is the same for each individual price increase.

Proof of Proposition 2

We show here that in the symmetric case with $k=0$, the Actual Loss as a fraction of initial sales is at least $(1-A^*)/m$.

We define δ as the diversion ratio between any pair of products, i.e., the fraction of sales lost by one product when its price alone rises that are captured by the other product. We assume that all the diversion ratios are constant in the relevant range and that each firm's demand is linear in price over the relevant range. So we can write the unit sales of Product 1 as $x_1 = \overline{x}_1 - (p_1 - \overline{p}_1) + \delta \sum_{j \neq 1} (p_j - \overline{p}_j)$. The Aggregate Diversion Ratio is $A = \delta(K-1)$ where there are *K* products. If the prices of all products other than Product 1 are the same, we can write $x_1 = \overline{x}_1 - (p_1 - \overline{p}_1) + A(p_2 - \overline{p}_2)$. We also can write the change in output at the other firms when Product 1's price alone changes as $x_2 = \overline{x}_2 - (p_2 - \overline{p}_2) + \delta(p_1 - \overline{p}_1) + \delta \sum_{j \neq 1,2} (p_j - \overline{p}_j)$. With symmetry among all of the other firms, this becomes $x_2 = \overline{x}_2 - (p_2 - \overline{p}_2)[-1 + A - \delta] + \delta(p_1 - \overline{p}_1)$.

Now we are ready to consider an arbitrary exogenous increase in p_1 and see how the outputs of all firms respond. Call $p_1 = \overline{p}_1 + h$ and $p_2 = \overline{p}_2 + \lambda h$ where this defines the matching rate $\lambda \leq 1$. So, we get $x_1 = \overline{x}_1 - h + A\lambda h$. The loss of sales at Product 1 are $\overline{x}_1 - x_1 = h(1 - \lambda A)$, so the loss rate is $g^* = 1 - \lambda A$. The gain in sales at Product 2 is $x_2 - \overline{x}_2 = (p_2 - \overline{p}_2)[-1 + A - \delta] + \delta(p_1 - \overline{p}_1)$ which can be written as $x_2 - \overline{x}_2 = \lambda h[-1 + A - \delta] + \delta h$. The total gain in sales by all of the other firms thus equals $X_{-1} - \overline{X}_{-1} = (K-1)h\{\lambda[-1 + A - \delta] + \delta\}$.

The Residual Aggregate Diversion Ratio is defined as

$$A^* \equiv \frac{X_{-1} - \overline{X}_{-1}}{\overline{x}_1 - x_1} = (K-1)\frac{\lambda[-1 + A - \delta] + \delta}{(1 - \lambda A)}.$$

Writing this in terms of primitives gives

$$A^* = (K-1)\frac{(\delta - \lambda) + \lambda\delta(K-2)}{1 - \lambda\delta(K-1)}.$$

Note as a check that if $\lambda = 0$ this does give back $A^* = \delta(K-1) = A$. Note also that with two firms, $K = 2$, we get $A^* = \frac{\delta - \lambda}{1 - \lambda\delta}$. We focus on the case where $A^* > 0$ which is equivalent to $\lambda(1 - \delta(K-2)) < \delta$.

Next, we measure the Actual Loss for the hypothetical monopolist who raises prices of all products uniformly by an amount *h*. Product 1's lost sales are $\overline{x}_1 - x_1^* = h(1 - \delta(K-1))$, which equals $h(1-A)$. Total lost sales are thus $Kh(1 - \delta(K-1))$. Note that the hypothetical monopolist's proportional losses are the single firm loss rate (unity with our normalizations) times one minus the aggregate diversion ratio.

Our method of using revealed-preference information requires that we compare the Actual Loss by the hypothetical monopolist with the loss that a single firm would incur given equilibrium matching by its rivals. We explore using the following *estimate* of the sales lost on each product by the hypothetical monopolist: $(\overline{x}_1 - x_1)(1 - A^*)$. This is the loss of sales at Product 1 if it raises its price and others follow, which is $h(1-\lambda\delta(K-1))$ times $1-A^*$.

Using this estimate, estimated total lost sales are $K(\overline{x}_1 - x_1)(1-A^*)$. Using $\overline{x}_1 - x_1 = h(1-\lambda A)$ from above, this estimate equals $Kh(1-\lambda A)(1-A^*)$. Since $A = \delta(K-1)$, this estimate is $Kh(1-\lambda\delta(K-1))(1-A^*)$.

This will *over-estimate* lost sales if and only if $Kh(1-\lambda\delta(K-1))(1-A^*) > Kh(1-\delta(K-1))$, which becomes $[(1-\lambda\delta(K-1)](1-A^*) > 1-\delta(K-1)$. Expanding the left-hand side, this becomes $1-\lambda\delta(K-1)-[1-\lambda\delta(K-1)]A^* > 1-\delta(K-1)$ or $(K-1)\delta(1-\lambda) > A^*[1-\lambda\delta(K-1)]$. This is the same as $\delta(1-\lambda) > \dfrac{A^*[1-\lambda\delta(K-1)]}{K-1}$. Substituting for the right-hand side using the definition of A^*, this becomes $\delta(1-\lambda) > (\delta-\lambda)+\lambda\delta(K-2)$. Simplifying, this becomes $1-\delta(K-1) > 0$, i.e., $\delta(K-1) < 1$: the Aggregate Diversion Ratio is less than unity.

Since we will over-estimate the lost sales by the hypothetical monopolist, if we find that the hypothetical monopolist earns greater profits by imposing the price increase than at the status quo, this comparison will certainly hold using the Actual Losses, which are lower than the estimated losses. Summarizing gives Proposition 3, a sufficient (but not necessary) condition for a group of products to form a market using the breakeven Critical Loss test.

Proof of Proposition 3

The asymmetry parameter k is defined so that the proportionate losses facing a single product that raises its price by a SSNIP are $(1+k)(s/m)$, where s is the chosen SSNIP (thus in general k will depend on the choice of SSNIP) and the (point) elasticity of demand facing the product is (by the Lerner equation) $1/m$. With this definition, the proportional loss of demand for the hypothetical monopolist following a SSNIP is $(1-A)(1+k)(s/m)$. Thus the Actual Loss is less than the Critical Loss if and only if $(1-A)(1+k)(s/m) \le s/(m+s)$. Solving for *A* gives

$$A \ge \frac{\frac{s}{m+s}+k}{1+k}.$$

With $k=0$ this simplifies to $A \ge s/(m+s)$ as in Proposition 1. The larger is *k*, the higher must be the aggregate diversion ratio for the products to form a relevant market.

To illustrate, with $s = .05$ and $m = 0.45$, $s/(m+s) = .05/(.45+.05) = 0.1$, so a group of products forms a market if and only if $A \geq (0.1+k)/(1+k)$. As noted above, for $k = 0$, the test is $A \geq 0.1$. For $k = 0.1$, i.e., losses from price increases are 110% of corresponding gains from price decreases, the test becomes roughly $A \geq 0.18$. For $k = 0.2$, i.e., losses from price increases are 120% of corresponding gains from price decreases, the test becomes $A \geq 0.25$.

We can solve the above expression for k to calculate the critical asymmetry ratio, k^*, i.e., the minimum amount of asymmetry that would be necessary to lead to a different conclusion:

$$k^* = \frac{A - \frac{s}{m+s}}{1-A},$$

which gives Proposition 2. Again using $s = .05$ and $m = 0.45$, we have $s/(m+s) = 0.1$, so $k^* = (A-0.1)/(1-A)$. If $A = 0.2$, then $k^* = .125$. If $A = 0.4$, then $k^* = .5$.

Pass-Through Rates and Asymmetry of Demand Response

We begin by deriving an expression for the pass-through rate, *r*. Write the demand curve as $x = f(p)$ where $f(p)$ is smooth. The curvature of the demand curve can be measured using $e(p) = pf''(p)/f'(p)$. Profits are $\pi(p) = (p-c)f(p)$. Maximizing profits gives $(p-c)f'(p) + f(p) = 0$. Differentiating with respect to cost gives $[2f'(p) + (p-c)f''(p)]\frac{dp}{dc} - f'(p) = 0$ so the pass-through rate is given by $r \equiv \frac{dp}{dc} = \frac{f'(p)}{2f'(p)+(p-c)f''(p)} = \frac{1}{2+(p-c)\frac{f''(p)}{f'(p)}} = \frac{1}{2+m\frac{pf''(p)}{f'(p)}}$. Therefore, we have $r = \frac{1}{2+me(p)}$ which can be written as $e(p) = \frac{1-2r}{rm}$.

We now relate *r* to the asymmetry parameter, *k*. By the definition of *k*, the sensitivity of demand to price for a SSNIP is $1+k$ times as large as the sensitivity of demand to price for a small reduction in price, so $f(\bar{p}+s\bar{p}) - f(\bar{p}) = (1+k)s\bar{p}f'(\bar{p})$, where $\bar{p}$ is the pre-merger price.

The second-order Taylor approximation to $f(\bar{p}+s\bar{p})$ is given by $f(\bar{p}+s\bar{p}) \approx f(\bar{p}) + s\bar{p}f'(\bar{p}) + \frac{(s\bar{p})^2}{2}f''(\bar{p})$, or $f(\bar{p}+s\bar{p}) - f(\bar{p}) \approx s\bar{p}f'(\bar{p}) + \frac{(s\bar{p})^2}{2}f''(\bar{p})$. (This approximation will be exact for quadratic demand curves, so we have allowed that degree of curvature, which obviously generalizes the linear demand case.) Using this approximation along with the equation defining *k* we have $(1+k)s\bar{p}f'(\bar{p}) \approx s\bar{p}f'(\bar{p}) + \frac{(s\bar{p})^2}{2}f''(\bar{p})$, which simplifies to

$kf'(\overline{p}) \approx \frac{s\overline{p}}{2} f''(\overline{p})$. Solving for *k* gives $k \approx \frac{s}{2}\frac{\overline{p}f''(\overline{p})}{f'(\overline{p})} = \frac{se(p)}{2}$. We are interested in the case of concave demand, so $f''(\overline{p}) \le 0$ which implies $e(p) \ge 0$ and $k \ge 0$. Using $e(p) = \frac{1-2r}{rm}$ gives

$$k \approx \frac{s}{m}\frac{1-2r}{2r}.$$

This is the expression reported in the text. We can also solve for r giving $r \approx \frac{1}{2}\frac{s}{s+mk}$.

Kinked Demand Curve Theory of Oligopoly

Suppose that rivals match price decreases at a rate λ_D and price increases at a rate $\lambda_I < \lambda_D$. A firm that decreases price by *h* gains sales of $h(1-\lambda_D A)$; a firm that increases price by *h* loses sales of $h(1-\lambda_I A)$. Therefore, $1+k = \frac{1-\lambda_I A}{1-\lambda_D A}$, so $k = \frac{(\lambda_D - \lambda_I)A}{1-\lambda_D A}$. With $\lambda_D = 1$ and $\lambda_I = 1/2$ this becomes $k = \frac{A}{2(1-A)}$. With $A = 0.2$ this gives $k = 1/8$.

Exclusion of Additional Benefit in the Profits of the Hypothetical Monopolist

If we do not include these benefits in the hypothetical monopolist's profits function, then the hypothetical monopolist has a marginal cost of *c* rather than $c - B$. Higher costs lead to higher prices, so this must lead to narrower markets than would inclusion.

Following the logic of Proposition 1, the point elasticity of demand facing a single product is now given by $\varepsilon = 1/(m+b)$. With linear demand, if a single firm raises the price of its product by a SSNIP, the percentage loss will be εs, which equals $s/(m+b)$. The Actual Loss for the hypothetical monopolist will be $(1-A)\varepsilon s$ which equals $(1-A)s/(m+b)$. Excluding the benefits from the hypothetical monopolist's profit function, the products form a relevant market if and only if this expression is less than the Critical Loss of $s/(m+s)$. This simplifies to $A \ge (s-b)/(m+s)$. With $b > s$ we get single-product markets.

Profit-Maximizing Price Increase vs. Break-Even Price Increase

With linear demand and constant marginal costs, profits are a quadratic function of price. It is a property of a quadratic profit function that the profit-maximizing price increase is half as large as the break-even price increase.