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# Tangent Point Orientation and Anticipated Trajectory Curvature - A Field Study on the Visual Control of High Speed Steering

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## Abstract

An important visual strategy in the visual control of driving during curve negotiation is tangent point orientation – directing gaze to the inside road edge in the area of interest (AOI) around the tangent point. Yet, while the phenomenon has been replicated in many studies at a qualitative level, and several computational models have been proposed to explain it, there is no consensus on whether the actual gaze target is the tangent point itself – or some other road point in its vicinity- and what the functional significance TP targeting (looking at the tangent point) or TP orientation (looking at a point in the TP AOI, but not necessarily the tangent point) might be. We report here a previously unobserved dependence between gaze distribution on road curvature: gaze concentrates on the part of the road where the vehicle yaw rate (local curvature) will be highest. We therefore suggest this geometric property of the future path may act as a functionally salient visual reference for the driver, and that the oft-reported “tangent point orientation” may in some cases be a side-effect caused by the spatial contiguity of the tangent point and the point of maximal path curvature.

**Keywords:** visually guided behavior, driving, eye-movements, field studies, tangent point, optic flow.

## Introduction

One of the ultimate goals in modeling eye-movements during natural behavior would be to be able to predict the whole sequences of eye movements executed during the performance of a naturalistic task, such as reading, car driving or preparing a meal (Kowler, 2011; Land 2006, 2007). This goal may not be as far off as it may seem, for as research on eye movements in naturalistic tasks during the past two decades has shown, in naturalistic tasks (as opposed to many artificial laboratory tasks) eye-movements present a picture of surprisingly stereotypical patterns, where the subjects’ gaze behavior is closely bound to the task conditions, both in spatial terms (gaze is concentrated only on task-relevant gaze targets) and in temporal terms (gaze target selection is closely coupled to the execution of different phases of a complex task, picking out targets “just in time” – i.e. selecting targets whose state needs to be verified, or which are about to be manipulated in 1-2s, so that the use of short term visual memory can be minimized). (Ballard et al., 1995). Successful modeling would thus entail (i) identifying the relevant gaze targets, (ii) specifying their relevance to the task (phases) in terms of the cognitive com-

putations the information provided by these targets could support, and (iii) testing the differential predictions of models against data collected from participants performing actual naturalistic tasks.

In the domain of car driving, one pattern in particular has been the subject of continuous research and theoretical debate, namely, *tangent point oriented curve negotiation* (Land & Lee, 1994): directing gaze in the direction of inside road edge during curve driving (Figure 1). Yet, while the basic phenomenon has been replicated in many studies (Underwood et al, 1999; Land & Tatler, 2001; Chattington et al., 2007; Kandil et al., 2009, 2010), and several computational models have been proposed to explain it (Land & Lee, 1994; Boer, 1996; Wann & Swapp, 2000; Wann & Land, 2000; Wann & Wilkie, 2004), there is no consensus on whether the gaze target is the tangent point itself – or some other road point in its vicinity- or the functional significance of this gaze pattern.

There are several models that account for the basic pattern (see Table 1). Some assume that the tangent point itself is the relevant gaze target for visually controlled steering (its eccentricity acting as a feedback control parameter), others that it is points on the forward-planned future trajectory which the driver looks at (only falling near the tangent point due to geometric reasons).

As the models are all compatible with the general observation of “tangent point orientation”, but make subtly different predictions concerning the precise spatial and temporal dynamics of gaze, more detailed on-road data is required to arbitrate between the various models. We report here a previously unobserved dependence between gaze distribution on road curvature: gaze concentrates on the part of the road where the vehicle yaw rate (local curvature) will be highest.

We therefore suggest this geometric property of the future path may act as a functionally salient visual reference for the driver, and that “tangent point orientation” may in fact – at least in some cases - be a side-effect caused by the spatial contiguity of the tangent point and this other gaze target. (This highlights the inherent problems of using AOI based measures in naturalistic environments where the experimenter has no control over the overlap of the AOI’s of different gaze targets).

Table 1: “Tangent point orientation” models of visual control of steering.

i. Land & Lee (1994)	Drivers fixate the tangent point and use the visual angle of tangent point (or gaze) relative to the locomotor axis to judge the curvature of the bend. (This model makes no specific prediction about steering).
ii. Land & Lee (1994), Wann & Land (2000)	Drivers fixate the tangent point and actively steer so as to keep the visual angle of the tangent point (and gaze) at a <i>constant horizontal direction</i> .
iii. Wann & Land (2000), Wann & Wilkie (2004)	The driver fixates a target point on their future path (i.e. a point they wish to pass through) which is near the tangent point not necessarily the tangent point, and then steers so that <i>the fixated point sweeps from its initial offset to directly in front of the locomotor axis at a constant rate</i> ..
iv. Boer (1996)	A reference point (“next to the tangent point but slightly into the road”) is chosen, although not necessarily fixated directly. Steering and speed are controlled in the following manner: The driver observes the visual angle between the vehicle’s heading and the target point, estimates the vehicle’s speed and the geometric distance to the target point, and adjusts steering and speed so that the following constraints are satisfied (1) the visual angle to the target point shall reach zero in less time than it will take to traverse the distance to the target point (2) the trajectory minimizes the maximum required lateral acceleration (i.e. minimizes the maximum steering input), and (3) the furthest deviation from the road edge remains within lane boundaries. (Active model).
v. Kim & Turvey (1996), Wann & Swapp (2000)	Visual flow is used to steer the car on a linear or locally circular trajectory: the driver fixates a target point on the road she wishes to pass through - a reference point on the future trajectory that is at rest in the allocentric reference frame, but moves in the egocentric frames of reference due to optic flow – and steers so that the visual flow lines are straight rather than curved. What is more, all those flow lines that fall on the observer's future path will now be vertical

## Subjects

Nineteen subjects participated in the experiment (11 male, 8 female, age range 23-52 years, mean 30, s.d. 7 years. All had held a valid driving license, and could be considered experienced drivers. Participants were recruited through university e-mail lists, some through personal contacts among students and staff. Condition for inclusion in the experiment was normal uncorrected vision (qualified to drive a car without correction) and sufficient driving experience (>20 000km). All participants were naïve to the purpose of the study (the tangent point hypothesis) and were given two cinema tickets as compensation for participating. All participants gave written informed consent, and the study was approved by the local ethics committee.

## Procedure

The test road was a 3.9 km low-standard two-lane rural road with a low traffic density and no lane markings (5.4 m pavement width). All drives were carried out in daylight. Participants drove the car to the test route, which was located 34 km from the campus. A few kilometers before arriving at the test road, the instrument panel was occluded. This was in done to reduce downwards glances to the speedometer during the test run, giving us the best opportunity to record road-directed gaze patterns. The participants did not express discomfort at having to drive without a speedometer. In addition to the participant who drove the car, a member of university staff acted as driving instructor on the front seat, giving route directions and ensuring safety.

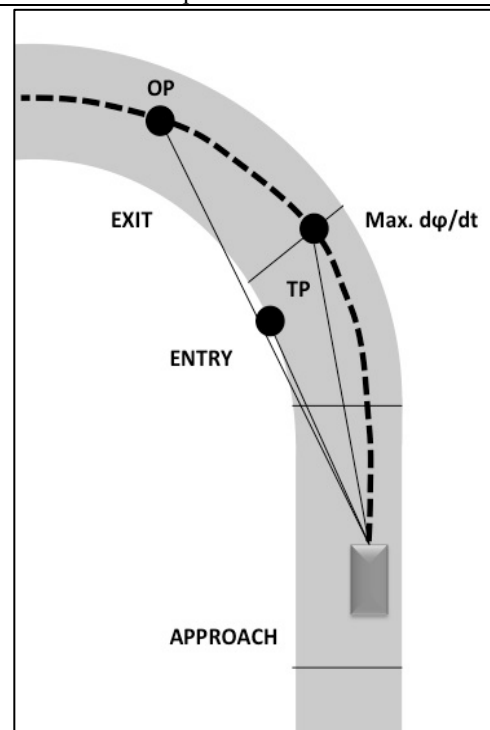


Figure 1: Illustration of some possible gaze targets in curve driving, discussed in the text. The future path of the vehicle is indicated by dotted line TP = tangent point; OP = occlusion point; point of maximum yaw rate is the point where the driver can begin to unwind the steering and accelerate out of the turn.

A research assistant acted as experiment instructor on the back seat on the driver's side. She was there to administer a cognitive secondary task on a different road section, the results of which are reported in Lehtonen, Lappi & Summala (2011). During the recording segments from which data is reported here the driver and the other persons in the car avoided any interaction. The participants drove the test route at their own pace. The driver was simply instructed to (i) drive as they normally would, but (ii) observe traffic laws and safety - in particular, they were explicitly instructed not to cut onto the lane of oncoming traffic in left-hand turns, if this was what they would do in normal driving. (This was deemed a necessary precaution because of the relatively high speed limit – 80 km/h – and the fact that many of the turns had a blind entry).

### Equipment and data preparation

The instrumented car was a model year 2007 Toyota Corolla 1.6 compact sedan with a manual transmission. The passenger side was equipped with brake pedals and extra mirrors for the driving instructor, as well as a computer display that allowed him to monitor vehicle speed, as well as the operation of the eye-tracker and the data-logging systems. The car was equipped with a two-camera (Smart Eye Pro versions 5.1 and 5.5 [www.smarteye.se](http://www.smarteye.se)) eye tracker operating at 60 Hz, a forward looking VGA scene camera, a GPS-receiver, as well as a forward looking infrared rangefinder (IBEO, [www.ibeoa-as.com](http://www.ibeoa-as.com)). Vehicle speed, the vehicle control signals (steering, throttle and brakes), as well as vehicle yaw rate were recorded directly from CAN-bus (all oversampled at 100 Hz). All signals were synchronized and time stamped on-line, and stored on a computer running custom MATLAB software, located in the rear luggage compartment. All subsequent data preparation, visualization and analysis was done with custom-made Python scripts, except for the final statistical analyses which were done with SPSS 18.

The data was segmented based on the time stamps corresponding to the GPS coordinates of the test route. To render different trials (drives) comparable, the data was given a distance-based representation. One trial, with no traffic or other “incidents” was chosen as a reference. The vehicle trajectory in an allocentric xy plane was computed by interpolating the GPS signal. This interpolated trajectory would then be used as the basis of a route-distance value. All participants' trials were then mapped onto this frame of reference, by first best-matching the observed GPS values to the reference trajectory, and then associating the rest of the data, with time-stamps matching the relevant GPS location.

The trajectory of the vehicle was also computed for individual trials by starting from point of gaze observation on the trajectory, and integrating the vehicle yaw rate and speed over time. This was used to estimate the point of gaze landing on the future trajectory, by finding first the point on the trajectory where the eccentricity of a point on the path-integrated trajectory corresponded with the visual eccentricity of gaze at the initial point – i.e. the point where the

line of sight would intersect the path-integrated trajectory. This gaze-landing point could then be assigned a route distance value based on the mapping of time stamps to route distances<sup>1</sup>

Tangent points were manually identified from still video frames from SmartEye's Scene camera (5 Hz frame rate), and the image coordinates of the mouse pointer and the eye-tracker coordinates were physically calibrated, using a calibration grid visible in the scene camera and the infrared rangefinder (whose coordinates were used as the native coordinate system for the car). The scene images were then associated with the rest of the data based on the time stamp of the video frame. If there was oncoming traffic in a turn (any vehicles or pedestrians visible in the forward-looking video camera), data for that turn was excluded from gaze-direction analyses (but included in the driving speed analyses). This was done in order to eliminate the effect of these potentially confounding visual targets in the road scene.

### Results

Four turns from the test route were selected for detailed analysis. The turns were chosen so that we would have two pairs of roughly similar turns, this way we could check whether any pattern of visual behaviour seen in a turn would also be seen in the other, similar, turn. The analysed turns comprised of two long left hand turns (hereafter denoted by T1 & T4), and two blind right-left sequences (T2/3 & T5/6)<sup>2</sup>. The blind sequences T2/3 and T5/6 probably resembled the roads used in Land and Lee (1994), while the faster turns T1 and T3, with a sighted approach phase, were probably more similar to the curves in Kandil et al. (2010).

#### Driver Gaze Behaviour in Relation to the Tangent Point (Tangent Point Orientation)

We first set out to replicate the commonly observed tangent point orientation. Based on the previous research reviewed above, we expected the drivers to direct their gaze towards the tangent point region, especially during the approach and

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<sup>1</sup> Note that the gaze-landing point, as defined here, may not in all cases perfectly coincide with the driver's true gaze target, because it does not use pitch-information and effectively projects the trajectory and gaze directions onto a two-dimensional plane; it does, however, provide a good estimate when the road is flat –as in the present experiment. The reason we did not use the pitch information is the difficulty of measuring it reliably in the noisy environment.

<sup>2</sup> We refer to a turn as blind when both of the following conditions are met: (i) At no point during the approach phase – i.e. before the driver steers into the turn - is the exit of the turn or the exit of the entire sequence in the case of connected curves visible to the driver, and (ii) During the entire approach phase, the occlusion point falls within some angular threshold of the tangent point (the threshold used here was 10°). The *occlusion point* is defined the furthestmost part of the desired trajectory to which a continuous line of passage is visible, i.e. the point on the road where the driving line first disappears from view (Fig. 1, see Lehtonen, Lappi & Summala, 2011). Turns that are not blind by these criteria are said to be sighted.

turn entry phases. As table 2 shows, we could demonstrate a consistent pattern of tangent point orientation during curve approach and turn entry. This establishes for the curves analysed the basic pattern observed in many previous studies, which have reported drivers to direct their gaze into the tangent point region 60-70% of the time, when entering a bend (Land & Lee, 1994; Kandil et al., 2010).

Table 2: Estimated relative frequency of tangent point oriented gazes (% of all valid observations, mean and standard deviation) falling within three degrees horizontal of the tangent point during turn approach (before the driver turns the wheel) and entry into the turn (after the driver turns the vehicle). In connected curve sequences only the first turn of the sequence for which an approach phase can be defined is listed.

	<i>Approach</i>	<i>Entry</i>
T1	52 (23)	41 (28)
T2/3	72 (24)	67 (22)
T4	50 (25)	58 (24)
T5/6	71 (25)	60 (22)

### Tangent Point and Gaze Behavior in Relation to the Vehicle Frame of Reference

We next set out to investigate further the behaviour of the tangent point and the driver’s gaze in vehicle coordinates. The Land & Lee (1994) model which predicts that the drivers steer so as to maintain the tangent point at a constant bearing angle. The Wann and Land (2000) model, in turn, predicts that tangent point eccentricity should decrease during turn entry (from a relatively eccentric value) when the driver begins to rotate the vehicle and to orient it towards a reference point (close to the tangent point) which he wishes to travel through.

Neither pattern was observed. Instead, as illustrated in figure 2, the observed horizontal positions of gaze and the eccentricity of the tangent point become progressively *more* eccentric during the turn-in phase of this right-left turn. The first vertical green line indicates the average point where the vehicle rotation rate exceeds one degree per second, the second indicates the point where the vehicle reaches its maximum rotation rate (the driver begins to turn the wheel back to the left) and the third green line indicates the point of maximum rotation-rate in the left hand turn (after which the driver begins to ea. reaching a maximum at towards the end of the turn-in phase.

Table 3 gives the values for tangent point displacement from the turn-in to the end of the entry phase. Although during the entry phase gaze is relatively concentrated in the TP region, the drivers nevertheless do not appear to be steering to compensate for the outward movement of the tangent point (and gaze) due to visual flow.

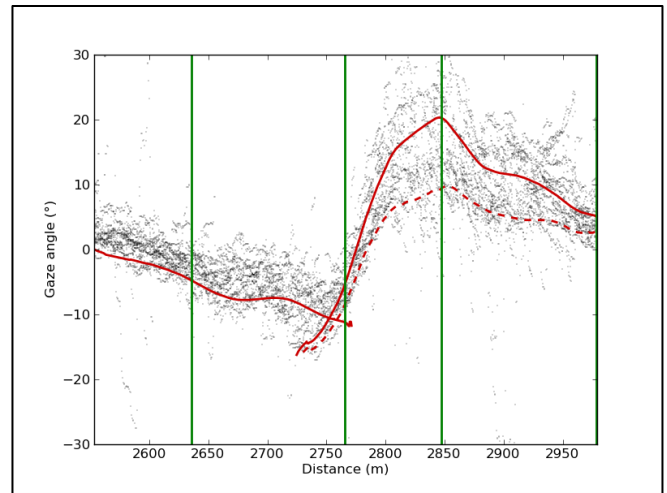


Figure 2: Tangent point and gaze in vehicle coordinates for the right-left turn sequence 5/6. The solid red line indicates the eccentricity of the tangent point (averaged across subjects), and the dotted line indicates the road centerline. Right is negative. Note that the tangent point (and gaze) are neither constant, nor do they sweep towards the vehicle centerline (zero eccentricity).

Table 3: Average displacement of the tangent point in the entry phase of each turn. Negative is to the right.

	N	mean
T1 entry	16	3.58
T2 entry	17	-8.68
T3 entry	16	25.1
T4 entry	16	0.66
T5 entry	15	-6.4
T6 entry	15	25.5

We could see the tangent point eccentricity (and gaze) was not constant through the turns but instead increased steadily, in step with the increase in vehicle yaw-rate as the car was entering the curve. As can be seen in Figure 2, the highest tangent point and gaze eccentricities appear occur systematically in the steepest parts of the turn (the green vertical lines indicating the parts of the trajectory where the yaw-rate values reach their maxima). This suggested that a possible relation might exist between the driver’s gaze and the vehicle yaw-rate. We decided to investigate this relationship by looking at the estimated fixation density on the road, i.e. the distribution of gaze landing points where the line of sight would intersect the vehicle trajectory.

The relative frequency of gaze landings in each 10 m road segment through each curve was computed as the percentage of all gaze landings falling within the segment. This was computed first individually, and then averaged across

subjects. This distribution is shown by the histogram in figure 3 (bottom), and shows that gaze is not equally distributed on the road surface. This would happen if the drivers were, for example, fixating a point on their trajectory that would be some constant distance ahead. Instead, as shown in the top half of figure 3, while they are approaching a turn the drivers are looking further ahead – 100-150 meters up the road in this instance – but as they enter the turn, the gaze falls closer to the car. Also, when in a connected sequence of turns, the gaze moves discontinuously on the road. Here, the gaze is seen to jump further up the road (from right to left) just before the vehicle yaw rate has reached its peak and the left-hand part of the turn begins.

An unexpected pattern became evident when the relative frequency of estimated gaze landings in a specific part of the curve was compared with yaw rate at that part of the road (histogram in fig.3, bottom shows data for one curve). Gaze landings appeared to be concentrating those parts of the road where the vehicle yaw rate (and thus the road curvature as well) would be highest. I.e. there is a high correlation between the vehicle yaw rate (local road curvature) and the frequency of gaze landings. This correlation is shown graphically in figure 4, and the numerical values of the correlations are given in Table 4, showing that drivers preferentially look at segments of the road where they are going experience a high yaw rate, i.e. parts of the road with high curvature.

Table 4: Correlation (Spearman’s Rho) between the median relative frequency (within a turn) with which a particular 10m road estimated as a potential gaze target, and the measured average yaw rate in that segment.

	n	Spearman’s Rho
T1	41	0.811
T2/3	46	0.565
T4	53	0.717
T5/6	43	0.795

### Discussion

There are many different steering models available and all predict the same qualitative pattern of tangent point orientation (i.e. orienting gaze in the general direction of the tangent point, the TP AOI), but different predictions concerning the details of the gaze distribution pattern.

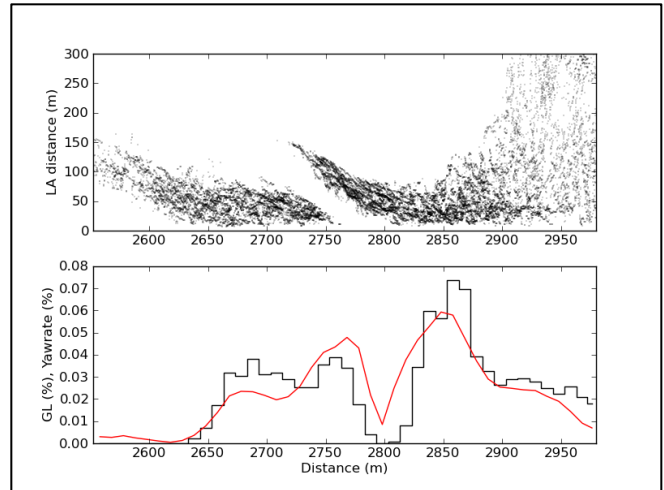


Figure 3: Horizontal axis location on the road (m from beginning of the route) Top: Estimated look-ahead distances through T5/6, i.e. distance of estimated gaze landing point from the current location, measured along the future path Bottom: Absolute value of vehicle yaw rate (continuous line) and the frequency histogram of gaze landings on each part of the curve (both variables scaled to sum to 100% within the analysed road section).

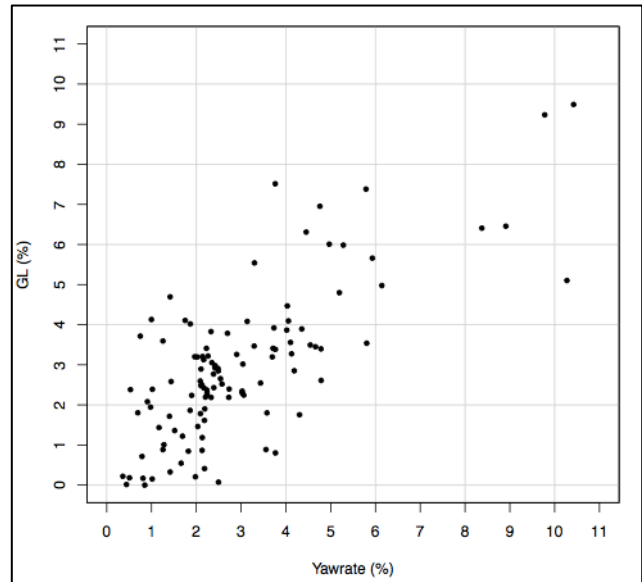


Figure 4: Top: Scatterplot of vehicle yaw rate and gaze landing density in the analysed turns. Each datapoint represents the across-subjects average for one 10m segment of all the curves analysed.

One model (Land & Lee 1994 / Wann & Land, 2000) would predict a constant value for gaze and tangent point eccentricity throughout a turn since, according to the model, the driver would make compensating steering movements (induce vehicle yaw) to cancel out the apparent horizontal motion of the tangent point due to optic flow. The prediction of the Wann and Land (2000) / Wann & Wilkie (2004) visual sweep model is that tangent point - and gaze - eccentricity should decrease during turn entry as the driver rotates the vehicle towards the reference point on the future path near the tangent point. Instead, we find that the tangent point - and gaze - become progressively *more* eccentric during the turn-in phase, without compensation or any apparent adverse effects to steering. We therefore conclude that these two steering models *which are based on the idea of using the tangent point as a visual target point which acts control parameter in set-point feedback loop* do not offer a general account for gaze/ steering behavior, at least on the kinds of turns analysed here (variable-radius curves on rural roads).

The differences of the remaining models - not based on simple negative feedback but on an *active trajectory planning strategy because they involve a predictive model of future path* – cannot be judged on the present data. It is thus not possible to pick one that would be the clear best fit for our data. However, the surprising finding that gaze was heavily concentrated on a few parts of the turn – namely, the locations where the vehicle achieved highest yaw-rate – offers some interesting possibilities. *Prima facie* none of the steering models appear to predict this high correlation between the yaw rate and probability of being selected as a gaze target, but we would consider that this pattern may be best interpreted as complementing the Wann & Swapp (2000) model or the Boer (1996) model. Both models predict that the driver should fixate *some* part of the road which he wishes to pass through, but neither specifies *which* part of the future trajectory the driver would or should look at. The location of highest anticipated trajectory curvature (max. yaw-dot) could perhaps serve as such a salient point of reference. (Note that is also behaviourally meaningful in terms of the sequencing of the driving task: this is the point where the curve/trajectory “opens up”, and the drive may begin to accelerate and unwind the steering).

There clearly exists a need for new and such accurate data on visual behaviour in curve driving – i.e. data that could speak the issue of which model or models offer the best fit to driver behavior. This study takes one steps in this direction, by presenting data that speaks directly to the *differential* predictions the models make beyond the *common* prediction of tangent point orientation (which was observed in all turns analysed). We also observed a surprising correlation between gaze landings on the road and measured vehicle rotation at that point, not predicted by any of the existing models.

It is suggested that the next generation of models be developed and empirically tested based on such detailed

quantitative basis - not only for the qualitative pattern of tangent point orientation.

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## References

- Ballard D, M Hayhoe & J Pelz (1995). Memory representations in natural tasks. *J cogn neurosci* 7: 66-80.
- Boer E R (1996) Tangent point oriented curve negotiation. Intelligent Vehicles Symposium, 1996, Proceedings of the 1996 IEEE: 7-12.
- Chattington M, M Wilson, et al. (2007) Eye–steering coordination in natural driving. *Exp Brain Res*,180: 1-14.
- Kandil F I, A Rotter, et al. (2009) Driving is smoother and more stable when using the tangent point. *J Vis* 9: 1-11. <http://www.journalofvision.org/content/9/1/11> doi:10.1167/9.1.11
- Kandil F I, A Rotter, et al. (2010) Car drivers attend to different gaze targets when negotiating closed vs . open bends. *J Vis* 10: 1-11.
- Kowler, E. (2011). Eye movements: the past 25 years. *Vis Res* 51: 1457-1483.
- Land, M F (2006) Eye movements and the control of actions in everyday life. *Prog Ret Eye Res* 25: 296-324.
- Land M F (2007) Fixation Strategies During Active Behaviour: A Brief History. In: R P G Gompel, M H Fishcer, W S Murray and R L Hill (eds) *Eye Movements: A Window on Mind and Brain*. Elsevier. Pp. 77-95.
- Land M F and D N Lee (1994) Where we look when we steer. *Nature* 369: 742-744.
- Land M F and B W Tatler (2001) Steering with the head: The visual strategy of a racing driver. *Curr Biol* 11: 1215-1220.
- Lehtonen E, Lappi O & Summala H (2011) Anticipatory eye movements when approaching a curve on a rural road depend on working memory load. *Trans Res Part F Traffic Psychol* (2011).
- Underwood G, P Chapman, et al. (1999) The visual control of steering and driving: Where do we look when negotiating curves? In: A.G.Gale, I.D.Brown, C.M.Haslegrave and S.P.Taylor (eds) *Vision in Vehicles - VII*. Amsterdam, Elsevier.
- Wann J and M Land (2000) Steering with or without the flow: is the retrieval of heading necessary? *Trends Cogn Sci* 4: 319-324.
- Wann J P and D K Swapp (2000) Why you should look where you are going. *Nat Neuroci* 3: 647.
- Wann J P and R M Wilkie (2004) How do we control high speed steering? In: L M Vaina, S A Beardsley, S K Rushton (eds.): *Optic flow and beyond*. Kluwer Academic Publishers. Norwell, MA, USA. pp. 371-389.